TEST PREP on Taylor Notes 1-3 - Dr. Loveless

Comments on Taylor Polynomials: Some things you may need in these sections:

- Taylor Notes - This is the department text for this topic. Read for theory, proofs and more examples.
- Full Review on Taylor Notes 1-3 - Dr. Loveless Full Review and Summary of TN 1-3 (see page 2 for how to do any error bound problem)
- Where we are going.... Taylor notes sections 1-4 are really about foundations and the process, then we learn the big facts in Taylor Notes 5 (these are facts you will often use beyond this course).
  - In Taylor Notes 1-3: We discuss error bounds only so that we can show that the approximations get better and better (and the error goes to zero) as we continue to the process.
  - In Taylor Notes 3-4: We use sigma notation and some helpful notational tricks to summarize the patterns we are seeing.

PARTICIPATION CODE: Don't forget to ask your TA for the participation code! Enter this on Test Prep Quiz for today on Canvas before the end of quiz section!

Winter 2018 - **Final Exam** (this was a full page)

8 Let $g(x) = \sqrt{3 + x^2}$

(a) Find the first Taylor polynomial based at $b = 1$.

(b) Use your answer to approximate the value of $\sqrt{3.25}$.

(c) Use Taylor's inequality to find an upper bound for the error of this approximation. (Use the interval $1/2 \leq x \leq 1$ since the approximation is when $x = 1/2$ and the polynomial is based at $b = 1$).

More examples of 'Given the interval, find the error'

Spring 2016 - Final (this was a full page)

8 Let $f(x) = e^{(x^2)}$

(a) Find the second Taylor polynomial based at $b = 1$.

(b) Find an upper bound for $|T_2(x) - f(x)|$ on the interval $[0, 2]$.

(c) What is the **smallest** value of $|T_2(x) - f(x)|$ on the interval $[0, 2]$?

Winter 2019 - Final (this was a full page)

7 Consider $f(x) = (1 + x)\sin(x)$.

(a) Find the second Taylor polynomial $T_2(x)$ based at $b = 0$.

(b) Use Taylor's inequality to find an error bound for $|f(x) - T_2(x)|$ on the interval $[-0.01, 0.01]$.

Examples of 'given the error, find the interval' - The method is to label the 'radius' of the interval, then solve for that unknown.

Spring 2019 - Final Exam (this was a full page, allow 12 minutes)

7 Let $f(x) = 1 + x + x^2 + 3x^3$

(a) Find the second-degree Taylor polynomial, $T_2(x)$ for $f(x)$ based at $b = 1$.

(b) Use Taylor's inequality to determine an interval around $b = 1$ on which $|T_2(x) - f(x)| < 0.024$.

Fall 2017 - Final (this was a full page)

7 Let $f(x) = \sqrt{x^2} = x^{3/2}$

(a) Find the second Taylor polynomial based at $b = 1$.

(b) Find an upper bound for $|T_2(x) - f(x)|$ on the interval $[1 - a, 1 + a]$ for $0 \leq a < 1$.

(c) Find a value of a such that $|T_2(x) - f(x)| \leq 0.004$ for all x in the interval $[1 - a, 1 + a]$.

Examples of 'given the error and interval, finding n' - The method is trial-and-error, then spot the pattern. Start with finding the error for $n = 1$, $n = 2$, and $n = 3$, then spot the pattern...

Spring 2017 - Final (this was a full page)

9 Let $T_n(x)$ be the n^{th} Taylor polynomial for $f(x) = e^{3x}$ based at $b = 0$. Find any value of n such that

$$|f(x) - T_n(x)| \leq 0.01$$

for all x in the interval $\left[-\frac{1}{3}, \frac{1}{3}\right]$.

Fall 2018 - Final (this was a full page)

9 Let $g(x) = e^{x/2}$.

(a) Find the third Taylor polynomial, $T_3(x)$, for $g(x)$ based at $b = 1$.

(b) On the interval $I = [0, 2]$, for which of the values of n below does the Taylor's inequality guaranteee that $|f(x) - T_n(x)| < 0.001$? You **must** show enough bound calculations to justify your answer.

Circle ALL that apply: $n = 2$ $n = 3$ $n = 3$ $n = 4$ $n = 6$