TEST PREP on 14.7 - Dr. Loveless

Engagement Reminder: Talk to each other, share ideas, relax, commiserate, and try to enjoy this time to work through problems. There is a real joy in problem solving and figuring things out, embrace these 50 minutes, keep asking questions and use your excellent TA to help you explore these ideas.

Test Prep Reminder: If you find it hard to find the joy in problem solving, no worries, instead focus on the practicality of the fact that you need to know this material to do well on the exams! These problems are *directly* from old exams. Keep asking yourself: Could I do this on a test? Can I do a page of an old exam in under 15 minutes? Do I know how to check my work? This self-assessment is critical, we can't read your mind and we can't tell you when you understand, so keep asking do I understand this? And ask questions of classmates and your TA if you don't *fully* understand something.

Focus on the first couple problems, then more if there is time. Also let your TA know if you have homework questions and you can discuss those as well as time allows. Use any problems you don't get to for study later (or you can bring them up at the next quiz section).

14.7 Extra Help: Here are a few of my review materials on this topic:

- Calculus 1 and 3 Max/Min Summary - One of my favorite review sheets that I have made. The first page is a full review of all the calculus 1 ideas on critical numbers and max/min (things you should already know) and the second page is a full review of all the calculus 3 ideas on critical points and max/min (the new stuff you are learning). I find it helpful to look at the one variable and two variable ideas side-by-side. Nice to see what is the same and what is different.
- Section 14.4 and 14.7 Review - Contains summaries and full examples of the main ideas in these sections. I do a long global max/min problem on the last page with all the details so you can see another example of how these problems are structured (that one is harder than any in homework).
- Chapter 14 Summary - One-page summary of chapter 14.

Winter 2016 - Exam 2 - Dr. Loveless (Classify critical points - Local Max, Local Min, Saddle Points).

2. (10 pts) Let $f(x, y) = 4xy - 3y + \frac{1}{x} - \frac{1}{4}\ln(y)$. Find and classify all the critical points of $f(x, y)$. **Clearly** show your work in using the second derivative test.

$$\left.\begin{array}{l} f_x = 4y - \frac{1}{x^2} \stackrel{?}{=} 0 \Rightarrow y = \frac{1}{4x^2} \\ f_y = 4x - 3 - \frac{1}{4y} \stackrel{?}{=} 0 \end{array}\right\} \text{COMBINE CONDITIONS!!}$$

$$4x - 3 - \frac{1}{4(1/(4x^2))} \stackrel{?}{=} 0 \Rightarrow 4x - 3 - x^2 = 0 \Rightarrow 0 = x^2 - 4x + 3 = (x-1)(x-3)$$

$$f_{xx} = 2/x^3, \quad f_{yy} = \frac{1}{4}\frac{1}{y^2}, \quad f_{xy} = 4 = f_{yx}$$

$$x = 1 \Rightarrow y = 1/4 \Rightarrow f_{xx} = 2,\ f_{yy} = 4,\ f_{xy} = 4 \Rightarrow D = (2)(4) - (4)^2 = -8 < 0$$

A SADDLE POINT OCCURS AT $(x,y) = (1, 1/4)$

$$x = 3 \Rightarrow y = \frac{1}{36} \Rightarrow f_{xx} = \frac{2}{27},\ f_{yy} = 324,\ f_{xy} = 4 \Rightarrow D = \frac{2}{27}324 - 4^2 = 8 > 0$$

$D+$, $f_{xx}+$, $f_{yy}+$ $\Rightarrow$ CONCAVE UP IN ALL DIRECTIONS

A LOCAL MIN OCCURS AT $(x,y) = (3, \frac{1}{36})$

More local max/min questions on this page...

Spring 2011 - Exam 2 - Dr. Loveless (this was a full page problem)

2. (9 pts) Let $f(x,y) = x^2y - x^2 - 2y^2$. Find and classify all critical points of $f(x,y)$. (Classify using appropriate partial derivative tests).

① $f_x = 2xy - 2x \stackrel{?}{=} 0 \Rightarrow 2x(y-1) = 0$ ← FACTOR FIRST!!!

② $f_y = x^2 - 4y \stackrel{?}{=} 0$

TWO CASES: $x=0$ OR $y=1$ NOW COMBINE!

① $x = 0 \xrightarrow{②} 0^2 - 4y = 0 \Rightarrow y = 0 \Rightarrow (x,y) = (0,0)$

① $y = 1 \xrightarrow{②} x^2 - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow x = -2 \Rightarrow (x,y) = (-2,1)$; $x = 2 \Rightarrow (x,y) = (2,1)$ } THREE CRITICAL POINTS

$f_{xx} = 2y - 2$, $f_{yy} = -4$, $f_{xy} = 2x = f_{yx}$

$\boxed{(0,0)} \Rightarrow f_{xx} = -2$, $f_{yy} = -4$, $f_{xy} = 0 \Rightarrow D = (-2)(-4) - 0^2 = 8 > 0$

$D+$, $f_{xx}\ -$, $f_{yy}\ - \Rightarrow$ CONCAVE DOWN IN ALL DIRECTIONS $\boxed{\text{LOCAL MAX}}$

$(-2,1) \Rightarrow f_{xx} = 0$, $f_{yy} = -4$, $f_{xy} = -4 \Rightarrow D = (0)(-4) - (-4)^2 = -16 < 0$

$(2,1) \Rightarrow$ SAME $\Rightarrow D = -16 < 0$

$\rightarrow$ $\boxed{\text{BOTH SADDLE POINTS}}$

Spring 2013 - Exam 2 - Dr. Loveless (this was half page problem)

1(b) (8 pts) Let $f(x,y) = \frac{9}{x} + 3xy - y^2$. Find and classify all critical points of $f(x,y)$. (Classify using appropriate partial derivative tests).

① $f_x = -\frac{9}{x^2} + 3y \stackrel{?}{=} 0 \Rightarrow y = 3/x^2$

② $f_y = 3x - 2y \stackrel{?}{=} 0$

COMBINE ① & ②: $\left(3x - 2\left(\frac{3}{x^2}\right) = 0\right) x^2 \xrightarrow{\text{CLEAR DENOM.!}} 3x^3 - 6 = 0 \Rightarrow x^3 = 2$

$x = 2^{1/3}$

ONLY ONE CRITICAL POINT.

$f_{xx} = \frac{18}{x^3}$, $f_{yy} = -2$, $f_{xy} = 3 = f_{yx}$

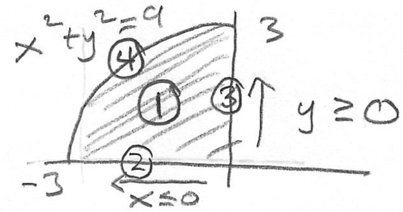
$x = 2^{1/3} \Rightarrow y = \frac{3}{2^{2/3}} = \frac{3}{2} \cdot 2^{1/3} \Rightarrow f_{xx} = \frac{18}{2} = 9$, $f_{yy} = -2$, $f_{xy} = 3$

$D = (9)(-2) - (3)^2 = -27 < 0$

A SADDLE POINT OCCURS AT $\boxed{(x,y) = \left(2^{1/3}, \frac{3}{2} \cdot 2^{1/3}\right)}$

READ THIS!!!!

These are global max/min questions...



Spring 2011 - Exam 2 - Dr. Loveless (this was half a page)

4. Consider the region $D = \{(x,y) \mid x \leq 0,\ y \geq 0,\ x^2+y^2 \leq 9\}$.

   (a) (7 pts) Find the absolute maximum and absolute minimum of $f(x,y) = yx^2 + 10$ over D.

★ [A SOL'N TO THIS PROBLEM MUST CLEARLY HAVE FOUR PARTS :

I CRITICAL POINTS INSIDE REGION?

(1) $f_x = 2yx = 0$, $f_y = x^2 = 0 \Rightarrow x = 0$ & $y =$ ANYTHING — ON BOUNDARY

NO CRITICAL PTS PROPERLY "INSIDE" REGION

II $y = 0,\ -3 \leq x \leq 0 \Rightarrow$ (2) $z = f(x,0) = 10$ ⟵ CONSTANT ⇒ MAX = 10, MIN = 10

III $x = 0,\ 0 \leq y \leq 3 \Rightarrow$ (3) $z = f(0,y) = 10$ ⟵ CONSTANT ⇒ MAX = 10, MIN = 10

ALSO CAN DO IN TERMS OF x

IV (4) $x = -\sqrt{9-y^2},\ 0 \leq y \leq 3 \Rightarrow z = f(-\sqrt{9-y^2}, y) = y(9-y^2) + 10$

$\Rightarrow z = 9y - y^3 + 10$ ONE VARIABLE PROBLEM!

$z' = 9 - 3y^2 = 0 \Rightarrow y = \pm\sqrt{3}$ — IN INTERVAL

ENDPTS ⟶ $y = 0 \Rightarrow z = 10$

$y = \sqrt{3} \Rightarrow z = 6\sqrt{3} + 10 \approx 20.39$

$y = 3 \Rightarrow z = 10$

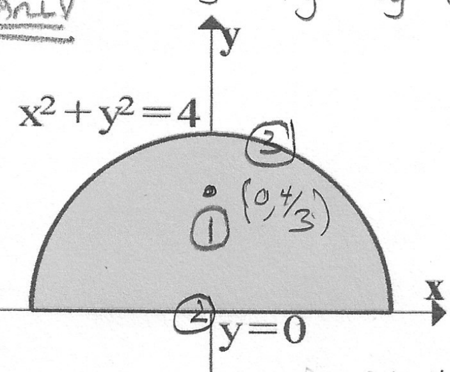
ABS. MIN = 10

ABS. MAX $= 6\sqrt{3} + 10 \approx 20.39$

Spring 2014 - Exam 2 - Dr. Loveless (this was a full page)

4. (14 pts) Find the absolute maximum and minimum values of $f(x,y) = y(x^2+y^2) - 2y^2 + 1$ over the region D shown below. $= yx^2 + y^3 - 2y^2 + 1$

★ [A SOL'N TO THIS PROBLEM MUST CLEARLY HAVE THREE PARTS.



I CRITICAL POINTS INSIDE REGION?

(1) $f_x = 2xy = 0 \rightarrow x = 0$ OR $y = 0$ — ON BOUNDARY!

$f_y = x^2 + 3y^2 - 4y = 0$

$x = 0 \Rightarrow 3y^2 - 4y = y(3y-4) = 0 \Rightarrow y = 0$ OR $y = 4/3$

$\boxed{(0, 4/3)}$ IS PROPERLY INSIDE REGION & IS A CRITICAL PT.

NOTE: $z = f(0, 4/3) = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 + 1 = -\frac{5}{27} \approx -0.19$

II (2) $y = 0,\ -2 \leq x \leq 2 \Rightarrow z = f(x,0) = 1$ ⟵ CONSTANT MAX = 1, MIN = 1

III (3) $x = \pm\sqrt{4-y^2},\ 0 \leq y \leq 2 \Rightarrow z = f(\pm\sqrt{4-y^2}, y) = 4y - 2y^2 + 1$

$z = 4y - 2y^2 + 1$ CALC. I PROBLEM! $z' = 4 - 4y = 0 \Rightarrow y = 1$

$y = 0 \Rightarrow z = 1$

$y = 1 \Rightarrow z = 4 - 2 + 1 = 3$

$y = 2 \Rightarrow z = 1$

OVERALL

ABS. MIN $= -\frac{5}{27} \approx -0.19$

ABS. MAX = 3

Applied max/min question...

Spring 2013 Honors - Exam 2 - Dr. Loveless (this was a full page)

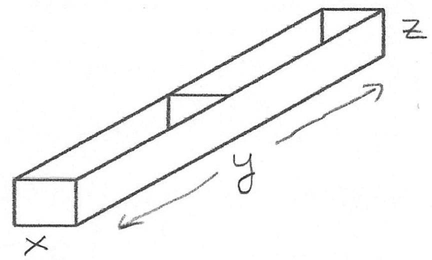
1. (13 pts)You are designing a cage to hold your pet rabbits. The cage is a rectangular box with a bottom, four sides, and one divider in the middle (you are keeping the males and females apart). There is no top. A picture of such a cage is below.

   If you want the total combined volume to be 12 cubic feet, then what dimensions will minimize the total material used?

   Give your final dimensions as decimals to four digits. As part of your answer, **use the second derivative test** to verify that your critical point is a local minimum.

SETUP

LABEL!



CONSTRAINT: VOLUME $= 12 \Rightarrow \boxed{xyz = 12}$

$\Rightarrow z = \dfrac{12}{xy}$

OBJECTIVE: MINIMUM MATERIAL (AREA)!

$$\text{MATERIAL USE} = \underbrace{xy}_{\text{BOTTOM}} + \underbrace{2yz}_{\text{SIDES}} + \underbrace{3xz}_{\text{FRONT/BACK/DIVIDER}}$$

USE CONSTRAINT TO MAKE OBJECTIVE A TWO VARIABLE FUNCTION!

$$f(x,y) = xy + 2y\left(\frac{12}{xy}\right) + 3x\left(\frac{12}{xy}\right) = xy + \frac{24}{x} + \frac{36}{y}$$

SOLVE

$f_x = y - \dfrac{24}{x^2} \stackrel{?}{=} 0 \Rightarrow y = 24/x^2$

$f_y = x - \dfrac{36}{y^2} \stackrel{?}{=} 0 \Rightarrow xy^2 - 36 = 0$

COMBINE CONDITIONS!

$x\left(\dfrac{24}{x^2}\right)^2 - 36 = 0 \Rightarrow \dfrac{24^2}{x^3} - 36 = 0 \Rightarrow 24^2 - 36x^3 = 0$

$\Rightarrow x^3 = \dfrac{24^2}{36} = \dfrac{4^2 \cancel{6^2}}{\cancel{6^2}} = 16$

$\Rightarrow x = (16)^{1/3} = 2(2)^{1/3} \approx 2.52$

$$\boxed{x = 16^{1/3} \Rightarrow y = \frac{24}{(16)^{2/3}} \Rightarrow z = \frac{12}{24/16^{1/3}} = \frac{1}{2}(16)^{1/3}}$$

$f_{xx} = 48/x^3, \quad f_{yy} = \dfrac{72}{y^3}, \quad f_{xy} = 1 = f_{yx}$

$\Rightarrow f_{xx} = \dfrac{48}{16} = 3, \quad f_{yy} = \dfrac{72}{54} = \dfrac{4}{3} \approx 1.33, \quad f_{xy} = 1$

$D \approx (3)(1.33) - (1)^2 = 3 > 0$

$D +, \; f_{xx} +, \; f_{yy} + \Rightarrow$ CONCAVE UP IN ALL DIRECTIONS

LOCAL MIN

$(x, y, z) \approx (2.51, 3.78, 1.26)$