Test Prep Solutions

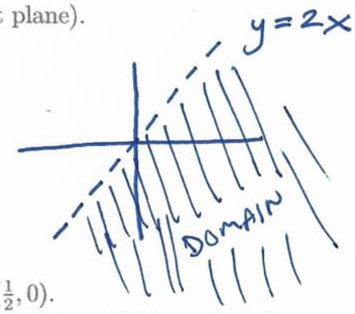
From Spring 2010, Exam 2, Loveless (domain, partial derivatives, tangent plane).

2. Let $z = f(x, y) = x\ln(2x - y) + x\cos(y^2)$

(a) Find and sketch the domain of the function, $f(x, y)$.

ONLY FUNCTION WITH RESTRICTION IS $\ln(2x-y)$, NEED $2x - y > 0$

$$\boxed{2x > y}$$



(b) Find the equation for the tangent plane to $f(x, y)$ at $(x, y) = (\frac{1}{2}, 0)$.
(For this test prep: Focus on finding f_x and f_y, then plug in the given point to each and see if you get the same thing as your classmates.)

SLOPE IN X-DIR

$$f_x = \overbrace{\ln(2x-y) + x \cdot \frac{2}{2x-y}}^{\text{PRODUCT RULE}} + \cos(y^2) \Rightarrow f_x(\tfrac{1}{2},0) = 0 + 1 + 1 = 2$$

$$f_y = x\,\frac{-1}{2x-y} + {}^{-}2y\,x\sin(y^2) \Rightarrow f_y(\tfrac{1}{2},0) = -\tfrac{1}{2} + 0 = -\tfrac{1}{2}$$

SLOPE IN y-DIR.

$f(\frac{1}{2},0) = 0 + \frac{1}{2}$ ← HEIGHT

$$z - \frac{1}{2} = 2\left(x - \frac{1}{2}\right) - \frac{1}{2}(y - 0)$$ ← TANGENT PLANE

2. The total surface area of a solid cone with radius r and height h is given by

$$A(r, h) = \pi r^2 + \pi r\sqrt{r^2 + h^2}.$$

$\sqrt{3^2+4^2} = \sqrt{25} = 5$

Find the equation for the tangent plane to $A(r, h)$ when $r = 3$ inches and $h = 4$ inches.

$$A_r = 2\pi r + \pi\sqrt{r^2+h^2} + \pi r\,\frac{\cancel{2}r}{\cancel{2}\sqrt{r^2+h^2}} \Rightarrow A_r(3,4) = 6\pi + 5\pi + \frac{9\pi}{5} = \frac{64\pi}{5}$$

$$A_h = 0 + \pi r\,\frac{\cancel{2}h}{\cancel{2}\sqrt{r^2+h^2}} \Rightarrow A_h(3,4) = \frac{12\pi}{5}$$

$$A(3,4) = 9\pi + 15\pi = 24\pi$$

TANGENT PLANE

$$\boxed{A - 24\pi = \frac{64\pi}{5}(r-3) + \frac{12\pi}{5}(h-3)}$$

THIS IS A GOOD WAY TO APPROXIMATE HOW SMALL CHANGES IN r OR h WILL EFFECT THE SURFACE AREA. THINKING OF A TANGENT PLANE AS AN APPROXIMATION TOOL IS CALL "LINEAR APPROXIMATION."

Problem below is from Fall 2013 - Exam 2 - Dr. Loveless (partial derivatives).

2(a) (6 pts) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ for $z^3 - 8z - e^2 = x^2\sqrt{y} + \ln(y) - e^{xy^3} - 1$ at the point $(x, y, z) = (2, 1, 3)$.

YOU SHOULD THINK X = INDEPENDENT, Z = Z(X) = DEPENDENT, Y = CONSTANT

$$\Rightarrow 3(z(x))^2 \frac{\partial z}{\partial x} - 8\frac{\partial z}{\partial x} - 0 = 2x\sqrt{y} + 0 - y^3 e^{xy^3} - 0$$

$$\Rightarrow (3z^2 - 8)\frac{\partial z}{\partial x} = 2x\sqrt{y} - y^3 e^{xy^3}$$

AT $(x, y, z) = (2, 1, 3)$, WE HAVE

$$(3\cdot(3)^2 - 8)\frac{\partial z}{\partial x} = 2(2)\sqrt{1} - (1)^3 e^{2(1)^3}$$

$$\Rightarrow 19\frac{\partial z}{\partial x} = 4 - e^2 \Rightarrow \boxed{\frac{\partial z}{\partial x} = \frac{4 - e^2}{19}} \approx -0.178$$