Name: ____________________ **Date:** ______________

Test Prep — Chapter 13 (Old Exam Pages) — Math 126

Participation: +1: show written 13.1 homework and +1: participate in this test prep

The problem below is from **Spring 2019 Exam 1 Page 3**. Start on your own for a few minutes, then discuss in groups, then check with your TA.

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3. For ALL parts below, consider the curve, C, given by $x = 5 - t$, $y = t$, $z = t^2 - 10$.

   (a) Find the **two** points (x, y, z) where the curve, C, intersects the cylinder $x^2 + y^2 = 13$.

   (b) Find parametric equations for the tangent line, L, to the curve, C, at $t = 1$.

   (c) Consider a different line L_2 given by $x = -2 + 6u$, $y = 2 + 4u$, and $z = 5 + 2u$. This line, L_2, and the curve, C, intersect in one point. Find the angle of intersection (round your answer to the nearest degree).

Next: If you are not done with your 13.1 and 13.2 HW, then switch to working on that now. If you are done, then continue on the following pages...

Spring 2023 — Exam 1 (Pages 3–4): *Some of these questions only require 13.1 and 13.2 and some require 13.3, can you see which you know how to start now?*

3. (12 pts)

   (a) Find the angle of intersection of the curves (Round final answer to the nearest degree)

   - $\mathbf{r}_1(t) = \langle t, 3-t, t^4 \rangle$
   - $\mathbf{r}_2(u) = \langle 2-u, 2u-2, 10-u^2 \rangle$

   Angle = ____________________

   (b) Let C be the curve of intersection of the surfaces

   $$y = \frac{1}{2}x^2 \quad \text{and} \quad z = \frac{1}{3}xy.$$

   Parameterize this curve, then use the parameterization to give the arc length from $(0,0,0)$ to $(3, \frac{9}{2}, \frac{9}{2})$.

   Arc Length = ____________________

4. (12 pts)

(a) True or False (circle one): *We will talk about this in 13.4*

i. TRUE   FALSE : $\mathbf{r}'(t)$ and $\mathbf{N}(t)$ are always orthogonal.

ii. TRUE   FALSE : $\mathbf{r}''(t)$ and $\mathbf{N}(t)$ are always parallel.

(b) Consider $\mathbf{r}(t) = \langle t^2, 3t + 6, -2t^2 \rangle$.

i. (5 pts) Find the curvature at $t = 0$. (Give your answer as a decimal rounded to three digits)

$\kappa(0) =$ ______________________

ii. (5 pts) Find the equation of the tangent line at the point $(4, 12, -8)$ and find where this line intersects the xz-plane.

Intersection with xz-plane: $(x, y, z) =$ ______________________

Chapter 12 and 13 Summary of Formulas

Vectors.

$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$	$c\mathbf{v} = \langle cv_1, cv_2, cv_3 \rangle$	$\frac{1}{\|\mathbf{v}\|}\mathbf{v}$ = unit vector in direction of $\mathbf{v}$
$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$	$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$	$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \cos(\theta)$	$\mathbf{u} \cdot \mathbf{v} = 0$ means orthogonal	θ is the angle if drawn tail to tail
$\mathbf{u} \times \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \sin(\theta)$	$\mathbf{u} \times \mathbf{v}$ is orthogonal to both	$\|\mathbf{u} \times \mathbf{v}\|$ = parallelogram area
$\text{comp}_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|}$	$\textbf{proj}_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2}\mathbf{a}$	

Comments: Know how to check/find vectors that are parallel or orthogonal. Be comfortable with computation, interpretations, and consequences.

Lines, Planes and Surfaces (assume the constants a, b and c are positive in the last three rows):

Lines: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$	(x_0, y_0, z_0) = a point on the line $\langle a, b, c \rangle$ = a direction vector
Planes: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$	(x_0, y_0, z_0) = a point on the plane $\langle a, b, c \rangle$ = a normal vector
Cylinder: One variable 'missing'	Know basics of traces
Elliptical Paraboloid: $z = ax^2 + by^2$	Hyperboloid Paraboloid: $z = ax^2 - by^2$
Ellipsoid: $ax^2 + by^2 + cz^2 = 1$	Cone: $z^2 = ax^2 + by^2$
Hyperboloid of One Sheet: $ax^2 + by^2 - cz^2 = 1$	Hyperboloid of Two Sheets: $ax^2 + by^2 - cz^2 = -1$

Comments: You should be very good at finding lines/planes and naming shapes.

Curves in $\mathbb{R}^3$:

$\mathbf{r}'(t) = \langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \rangle$	$\mathbf{r}''(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \rangle$
$\int \mathbf{r}(t)\,dt = \left\langle \int x(t)dt, \int y(t)dt, \int z(t)dt \right\rangle$	Note: There are three constants of integration.
Arc Length $= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt$	$s(t) = \int_0^t \sqrt{(x'(u))^2 + (y'(u))^2 + (z'(u))^2}\,du$
$\kappa(t) = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$	$\kappa(x) = \frac{\|f''(x)\|}{(1+f'(x)^2)^{3/2}}$ = 2D curvature
$\mathbf{r}'(t) = \mathbf{v}(t)$ = velocity vector	$\|\mathbf{r}'(t)\| = \|\mathbf{v}(t)\|$ = speed
$\mathbf{r}''(t) = \mathbf{a}(t)$ = acceleration	$\mathbf{r}(t) = \int \mathbf{v}(t)\,dt, \quad \mathbf{v}(t) = \int \mathbf{a}(t)\,dt$
$\mathbf{T}(t) = \frac{1}{\|\mathbf{r}'(t)\|}\mathbf{r}'(t)$ = unit tangent	$\mathbf{N}(t) = \frac{1}{\|\mathbf{T}'(t)\|}\mathbf{T}'(t)$ = principal unit normal
$\mathbf{B}(t) = \mathbf{T} \times \mathbf{N}$ = binormal	$\mathbf{r}'(t) \times \mathbf{r}''(t)$ = parallel to $\mathbf{B}$
$a_T = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{\|\mathbf{r}'(t)\|}$	$a_N = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|}$

Comments: You should be comfortable working with curves, finding intersections, finding tangent vectors, finding tangent lines, and computing curvature and arc length.