TEST PREP on 13.1 and 13.2 - Dr. Loveless

Test Prep Reminder: These problems mostly come *directly* from the Dr. Loveless old exam archive. You can find solutions in that archive after class. I give quite a few problems, you won't get to all of them, but use the others for check if you are ready for exam questions on 13.1/13.2.

13.1-13.2 Extra Help: Here are a few of my review materials on this topic if you need extra help:

- Section 13.1 Review - includes additional examples on visualizing, parameterizing an intersection, finding an intersection of curves, and what it means for two object in motion to collide.
- Section 13.2 Review - a basic fact sheet on tangent vectors, tangent lines and notation.
- Calculus Fact Sheet - Derivatives and integrals you can quote. You will have to do some derivatives in chapter 13 (and a few basic integrals... know substitution, by parts, and how to find "+C").

PARTICIPATION CODE: Don't forget to ask your TA for the participation code! Enter this on Canvas before the end of quiz section!

From Fall 2018, Exam 1, Loveless (quick "surface of motion" question)

1(a) Which of the following functions describe points that are always on the curve of intersection of the surfaces $x^2 + z^2 = 4$ and $x = 2y$: (select ALL for which every point on the curve is on the intersection).

(a) $\mathbf{r}_1(t) = \langle t, \frac{1}{2}t, \sqrt{4 - t^2} \rangle$

(b) $\mathbf{r}_2(t) = \langle 2\cos(t), \cos(t), 2\sin(t) \rangle$

(c) $\mathbf{r}_3(t) = \langle 2\sin(t^3), \sin(t^3), 2\cos(t^3) \rangle$

(d) $\mathbf{r}_4(t) = \langle 2t, t, 0 \rangle$

From Spring 2013 (Honors), Exam 1, Loveless (finding a tangent line... this was half a page).

4(a) Dr. Loveless has motion sickness. You trick him into getting on a roller coaster that follows the path given by the vector function: $\mathbf{r}(u) = \langle 20\sin(u), 24u, 20\cos(u) + 40 \rangle$. When the ride gets to the point $(x, y, z) = (10\sqrt{3}, 8\pi, 50)$, Dr. Loveless' calculator falls out of his pocket. Assume the calculator follows the path of the tangent line (we will assume there is no gravity, ha).

If the xy-plane is the ground, at what location (x, y, z) does the calculator land on the ground?

From Winter 2016, Exam 1, Loveless (finding a tangent line as well as intersecting curves).

4. At time $t = 0$, a egg is set in motion toward Dr. Loveless by a disgruntled student (Dr. Loveless is sitting on the xy- plane at the point where the egg will eventually hit him). The egg's path is described parametrically by $x = t$, $y = 8\sqrt{t+1}$, $z = 15t - t^2$.

   (a) At the time $t = 0$, find a vector that is tangent to the path of the egg and has length 5.

   (b) Find parametric equations for the tangent line at the positive time when the egg hits the xy-plane.

   (c) A less disgruntled student tries to save Dr. Loveless and set a rock in motion following the path given by $x = 3$, $y = 14 + t$, $z = 28 + t^3$. The paths of the rock and the egg do intersect, but they do NOT collide (unfortunately for Dr. Loveless). Find the (acute) angle of intersection of the paths of these two curves.
   Give your final answer in degrees rounded to two digits after the decimal. (Use different parameters!!)

From Spring 2014, Exam 1, Loveless (finding a tangent line as well as intersecting a curve and a surface).

4. You are sitting at the origin on the surface $4z - x^2 - y^2 = 0$. You launch a water balloon into the air and its position at time t seconds is given roughly by the vector function $\mathbf{r}(t) = \langle t, 2t, 20t - 5t^2 \rangle$.

   (a) Give the two word name of this surface, $4z - x^2 - y^2 = 0$.

   (b) Your math instructor just happens to be sitting at the location where the water balloon lands on this surface. Find the (x, y, z) location where your math instructors is sitting.

   (c) Find parametric equations for the tangent line to the path at $t = 2$.

   (d) Find the curvature at time $t = 2$. (This is from 13.3, ignore it until we cover 13.3