1. (12 points)

6 (a) Find the equation of the plane that contains the point $(1, -2, 3)$ and the line given by $x = 4t$, $y = 1 - t$, $z = 5 + 2t$.

NEED TWO VECTORS PARALLEL TO THE PLANE.

POINTS ON PLANE: $\underset{t=0}{A(0,1,5)}$, $\underset{t=1}{B(4,0,7)}$, $\underset{\text{given}}{C(1,-2,3)}$

VECTORS: $\underbrace{\vec{AB} = \langle 4,-1,2\rangle}_{\text{direction vector for line}}$, $\vec{AC} = \langle 1,-3,-2\rangle$

NORMAL: $\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 2 \\ 1 & -3 & -2 \end{vmatrix} = (2 - -6)\vec{i} - (-8-2)\vec{j} + (-12 - -1)\vec{k}$

$= \langle 8, 10, -11\rangle$

PLANE: $\boxed{\begin{gathered} 8(x-1) + 10(y+2) - 11(z-3) = 0 \\ 8x + 10y - 11z + 45 = 0 \end{gathered}}$

6 (b) Consider the line through the point $(0, 3, 5)$ that is orthogonal to the plane $2x - y + z = 20$. Find the point of intersection of the line and the plane.
(Hint: Start by finding parametric equations for the line).

LINE EQUATIONS:
$x = 0 + 2t$
$y = 3 - t$
$z = 5 + t$

NORMAL TO PLANE: $\underbrace{\langle 2,-1,1\rangle}_{\text{DIRECTION VECTOR FOR THE LINE}}$

INTERSECTION: $2x - y + z = 20$

$2(2t) - (3-t) + (5+t) = 20$

$\Rightarrow 4t - 3 + t + 5 + t = 20$

$6t = 18$

$t = 3$

$\Rightarrow \boxed{(x, y, z) = (6, 0, 8)}$

ASIDE: You could now find the dist. from $(0,3,5)$ to $(6,0,8)$ to get the distance to the plane.

2. (5 pts) Consider the surface $z = x^2 + 2y^2$.

(a) Describe the traces parallel to the given plane (no work needed, just circle your answers).

i. Parallel to the $yz$-plane (when $x$ is fixed):

PARABOLAS  CIRCLES  ELLIPSES  HYPERBOLAS  NONE OF THESE

ii. Parallel to the $xz$-plane (when $y$ is fixed):

PARABOLAS  CIRCLES  ELLIPSES  HYPERBOLAS  NONE OF THESE

iii. Parallel to the $xy$-plane (when $z$ is fixed, $z > 0$):

PARABOLAS  CIRCLES  ELLIPSES  HYPERBOLAS  NONE OF THESE

(b) Clearly circle the name of the surface given by $z = x^2 + 2y^2$:

| | | |
|---|---|---|
| CONE | SPHERE | ELLIPSOID |
| PARABOLIC CYLINDER | CIRCULAR CYLINDER | ELLIPTICAL CYLINDER |
| HYPERBOLIC CYLINDER | HYPERBOLOID | CIRCULAR PARABOLOID |
| ELLIPTIC PARABOLOID | HYPERBOLIC PARABOLOID | NONE OF THESE |