TEST PREP on 10.3/15.3 - Dr. Loveless

Notes on 10.3/15.3:

- Trig Fact Sheet - The Unit Circle and trig facts (put your calculator AWAY, it is faster to use this on homework, plus the homework wants exact answers)
- Polar Coordinates Review - Summary of polar facts/graphing.
- How to integrate Powers of Sine and Cosine - You'll need this in 15.3.

My polar graphing policy:

In 15.3 homework, you'll encounter 3-4 problems with loops or cardiods where they don't provide a picture. In those homework problems take some time to plot several points and get a rough picture. However, I want to give you some comfort that on my *exams*, my policy is this:

1. Be able to graph circles. That includes circles centered at places other than the origin.
2. Be able to plot a few key points (like intercepts or at a given angle value), without a picture.
3. If I give you something other than a circle (like a cardiod or loops), then *I will provide a picture on the exam* if a picture is somehow helpful in completing the problem.

About this test prep: The first three problems are on 10.3 only. Then the rest are from 15.3. Try to do the first couple in groups. Then perhaps try one from the 15.3 part. Also let your TA know if you have homework questions.

Spring 2013 - ***Exam 1*** - Dr. Loveless (plot points to match graphs) THINK INTERCEPTS!

3(b) In the blanks provided to the left of each graph, put the number of the polar equation that matches the graph in the xy-plane (two graphs will not be labeled).

1. $r = \sqrt{\theta}$

r increases as θ increases! SPIRAL

2. $r = 1 - 2\cos(\theta)$

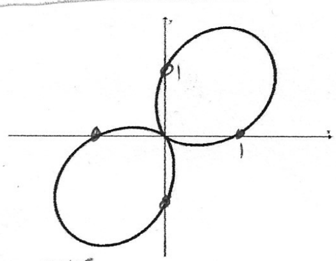
θ	0	$\pi/2$	π	$3\pi/2$
r	-1	1	3	1

3. $r = 1 + \sin(2\theta)$

θ	0	$\pi/2$	π	$3\pi/2$
r	1	1	1	1

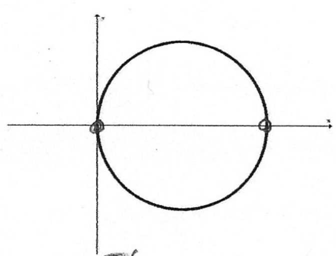
4. $r = 9\cos(\theta)$

θ	0	$\pi/2$	π	$3\pi/2$
r	9	0	-9	0



3

BY INTERCEPTS ONLY THAT MATCHES



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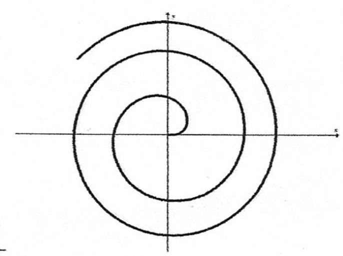
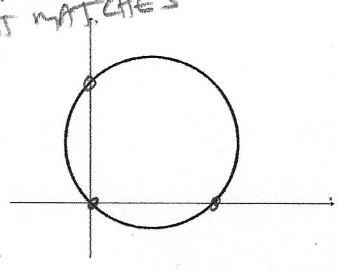


1



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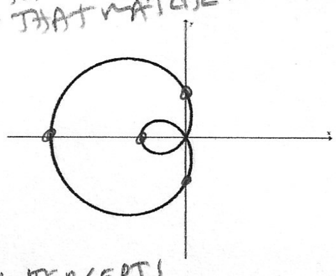
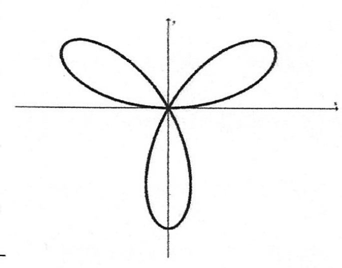


2

BY INTERCEPTS ONLY THAT MATCHES



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Spring 2014 - ***Exam 1*** - Dr. Loveless (plot points)

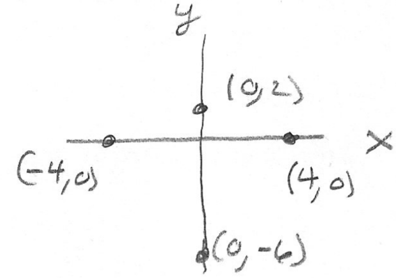
2(b) The polar curve $r = 4 - 2\sin(\theta)$ has exactly two x-intercepts and two y-intercepts. Give the (x, y) coordinates for all the intercepts.

$\theta = 0 \Rightarrow r = 4 \Rightarrow (x,y) = (4,0)$

$\theta = \pi/2 \Rightarrow r = 4-2 = 2 \Rightarrow (x,y) = (0,2)$

$\theta = \pi \Rightarrow r = 4 \Rightarrow (x,y) = (-4,0)$

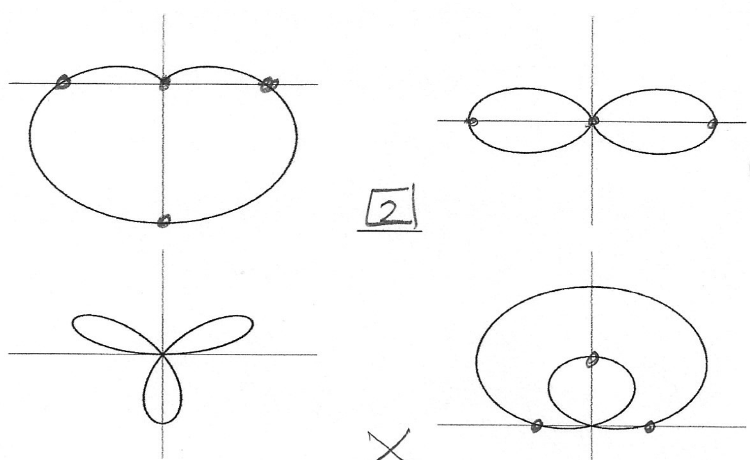
$\theta = \frac{3\pi}{2} \Rightarrow r = 4+2 = 6 \Rightarrow (x,y) = (0,-6)$



Spring 2010 - ***Exam 1*** - Dr. Loveless (plot points to match graphs).

3(b) Match the polar equations to the correct graphs. Put the number of the equation next to the correct picture in the blanks provided. No formal explanation is required. (There is one extra picture that won't be labeled).

1. $r = 3 - 3\sin(\theta)$
2. $r = 1 + \cos(2\theta)$
3. $r = \sin(3\theta)$



THINK INTERCEPTS FIRST!

1

θ	0	$\pi/2$	π	$3\pi/2$
r	3	0	3	6

2

θ	0	$\pi/2$	π	$3\pi/2$
r	2	0	2	0

3

θ	0	$\pi/2$	π	$3\pi/2$
r	0	0	0	0

CAN'T BE THIS ONE, SO HAS TO BE THE OTHER

The rest of this is 15.3 Examples..

complete square

$x^2 - 2x + 1 + y^2 = 0 + 1$

$(x-1)^2 + y^2 = 1$

Winter 2016 - Exam 2 - Dr. Loveless (a region between circles)

4(b) Let R be the region in the $\boxed{\text{first quadrant}}$ between the circle $\underbrace{x^2+y^2=9}_{r=3}$ and the circle $\underbrace{x^2+y^2=2x}$.

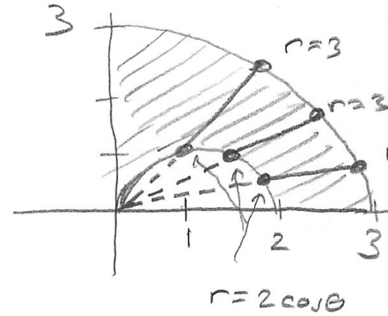
Using polar coordinates, evaluate $\displaystyle\iint_R \frac{y}{x^2+y^2}\,dA$.

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

(Note: I provided a picture on this exam, but you try to graph the region.. these are circles)

FOR $0 \le \theta \le \pi/2$, $2\cos\theta \le r \le 3$ $\longrightarrow$



$$\int_0^{\pi/2}\int_{2\cos\theta}^{3} \frac{r\sin\theta}{\cancel{r^2}}\,\cancel{r}\,dr\,d\theta$$

$$= \int_0^{\pi/2} \sin\theta \; r\Big|_{2\cos\theta}^{3}\, d\theta$$

$$= \int_0^{\pi/2} 3\sin\theta - 2\sin\theta\cos\theta\,d\theta$$

$u = \cos\theta$

$du = -\sin\theta\,d\theta$

$$= -3\cos\theta\Big|_0^{\pi/2} + \underbrace{\int_0^{\pi/2} -2\sin\theta\cos\theta\,d\theta}$$ $\longrightarrow$ $\displaystyle\int_1^0 2u\,du = u^2\Big|_1^0 = 0-1$

$= -1$

$$= (-3\cdot 0) - (-3\cdot 1) + -1 = \boxed{2}$$

Spring 2018 - Exam 2 - Dr. Loveless (graph the region first! Then set up in polar)

4(b) Rewrite the following double integral in polar coordinates, then evaluate:

$$\int_0^{\sqrt{2}}\int_y^{\sqrt{4-y^2}} x^2\,dx\,dy.$$

$0 \le y \le \sqrt{2}$

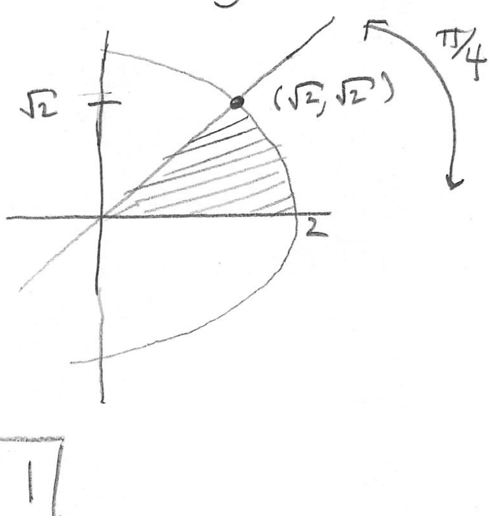
$y \le x \le \sqrt{4-y^2}$

$x = \sqrt{4-y^2}$ RIGHT

$x = y$ LEFT

$$= \int_0^{\pi/4}\int_0^{2} r^2\cos^2\theta \; r\,dr\,d\theta$$

$\frac{16}{4} = 4$



$$= \int_0^{\pi/4} \cos^2\theta \; \frac{1}{4}r^4\Big|_0^2\,d\theta$$

$$= 4\int_0^{\pi/4} \frac{1}{2}(1+\cos(2\theta))\,d\theta$$

$$= 2\left[\theta + \frac{1}{2}\sin(2\theta)\Big|_0^{\pi/4}\right]$$

$$= 2\left[\left(\frac{\pi}{4} + \frac{1}{2}\right) - (0+0)\right] = \boxed{\frac{\pi}{2} + 1}$$

Fall 2013 - *Exam 2* - Dr. Loveless (notice, I provided a picture!)

$z=1$

3. Find the area of the region outside the circle $x^2+y^2=\frac{1}{4}$ and inside one loop of the polar curve $r=\cos(2\theta)$.

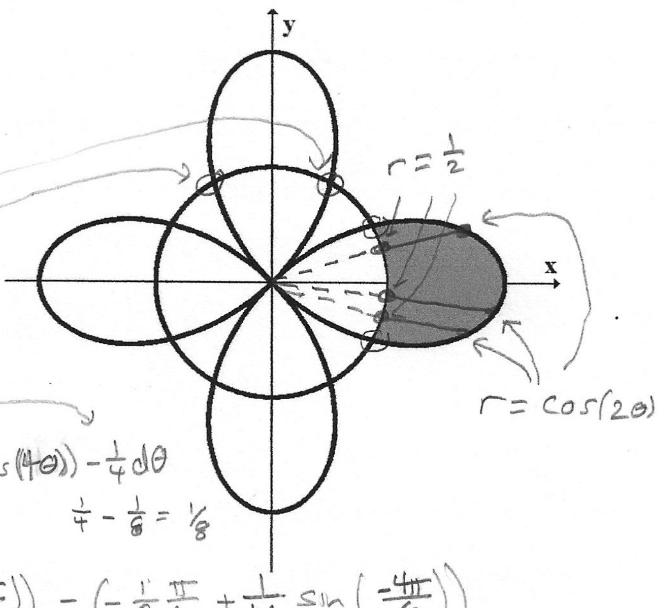
STEP 1 $\iint_R 1\,dA$

STEP 2 $x^2+y^2=\frac{1}{4} \Rightarrow r^2=\frac{1}{4} \Rightarrow r=\frac{1}{2}$ "INSIDE"

INTERSECT: $\frac{1}{2}=r\stackrel{?}{=}\cos(2\theta)$

$\Rightarrow 2\theta=\ldots,-\frac{\pi}{3},\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\ldots$

$\theta=\ldots,-\frac{\pi}{6},\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\ldots$



STEP 3

$$\int_{-\pi/6}^{\pi/6}\int_{1/2}^{\cos(2\theta)} 1\cdot r\,dr\,d\theta=\int_{-\pi/6}^{\pi/6}\frac{1}{2}r^2\Big|_{1/2}^{\cos(2\theta)}d\theta$$

$$=\frac{1}{2}\int_{-\pi/6}^{\pi/6}\cos^2(2\theta)-\frac{1}{4}\,d\theta=\frac{1}{2}\int_{-\pi/6}^{\pi/6}\frac{1}{2}(1+\cos(4\theta))-\frac{1}{4}d\theta$$

$\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$

$$=\int_{-\pi/6}^{\pi/6}\frac{1}{8}+\frac{1}{4}\cos(4\theta)d\theta$$

$$=\frac{1}{8}\theta+\frac{1}{16}\sin(4\theta)\Big|_{-\pi/6}^{\pi/6}=\left(\frac{1}{8}\frac{\pi}{6}+\frac{1}{16}\sin\left(\frac{4\pi}{6}\right)\right)-\left(-\frac{1}{8}\frac{\pi}{6}+\frac{1}{16}\sin\left(\frac{-4\pi}{6}\right)\right)$$

$$=\left(\frac{\pi}{48}+\frac{1}{16}\frac{\sqrt{3}}{2}\right)-\left(-\frac{\pi}{48}+\frac{1}{16}\frac{-\sqrt{3}}{2}\right)=\frac{2\pi}{48}+\frac{2\sqrt{3}}{16\cdot 2}=\boxed{\frac{\pi}{24}+\frac{\sqrt{3}}{16}}$$

Winter 2016 - *Final* - (notice I would provide a picture)

$z=1$

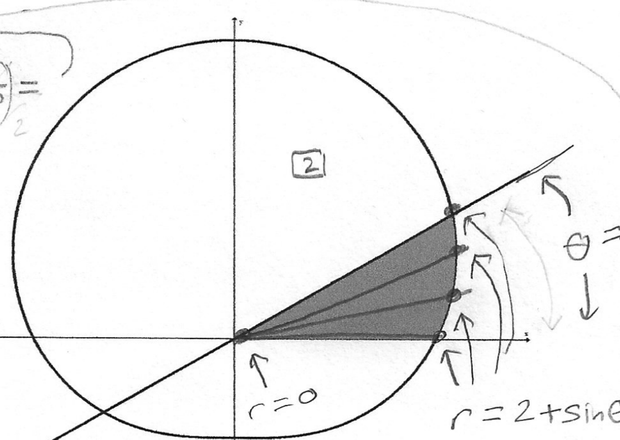
3. Find the area of the region bounded by the curve $r=2+\sin(\theta)$, the line $y=\frac{\sqrt{3}}{3}x$ and the x-axis (the region is shown below).

$\text{slope}=\frac{\text{RISE}}{\text{RUN}}=\tan\theta=\frac{\sqrt{3}}{3}$ SAME

WE SHOULD RECOGNIZE $\tan\left(\frac{\pi}{6}\right)=\frac{1/2}{\sqrt{3}/2}=\frac{1}{\sqrt{3}}=$

FOR ANY θ WITH $0\le\theta\le\pi/6$

WE HAVE $0\le r\le 2+\sin\theta$



1 $\iint_R 1\,dA=\int_0^{\pi/6}\int_0^{2+\sin\theta}1\cdot r\,dr\,d\theta$ 3

$$=\int_0^{\pi/6}\frac{1}{2}r^2\Big|_0^{2+\sin\theta}d\theta$$

$$=\frac{1}{2}\int_0^{\pi/6}(2+\sin(\theta))^2d\theta=\frac{1}{2}\int_0^{\pi/6}4+4\sin\theta+\sin^2\theta\,d\theta$$

$\frac{1}{2}(1-\cos(2\theta))$

4

$$=\frac{1}{2}\int_0^{\pi/6}4+4\sin\theta+\frac{1}{2}-\frac{1}{2}\cos(2\theta)d\theta$$

$$=\frac{1}{2}\left[\frac{9}{2}\theta-4\cos\theta-\frac{1}{4}\sin(2\theta)\Big|_0^{\pi/6}\right]=\frac{1}{2}\left[\left(\frac{9}{2}\frac{\pi}{6}-4\frac{\sqrt{3}}{2}-\frac{1}{4}\frac{\sqrt{3}}{2}\right)-(0-4-0)\right]$$

$$=\frac{1}{2}\left[\frac{3\pi}{4}-\frac{17}{4}\frac{\sqrt{3}}{2}+4\right]=\boxed{\frac{3}{8}\pi-\frac{17}{16}\sqrt{3}+2}$$