MATH 126 – CHAPTER 14 PRACTICE EXAM

Sample C

Your Name

Your Signature

About this practice exam

These problems are drawn from previous Math 126 exams, assigned WebAssign homework, and course worksheets/test preps. They have been reorganized into a complete practice exam. This is one of five practice exams designed to help you prepare for Exam 2. The questions and difficulty are *not* a prediction of the actual exam. Sources and related homework appear below each problem. Try the exam under timed conditions, then use the solutions to review your work.

Instructions

- CHECK that your practice exam contains 7 problems on 6 pages (3 double-sided sheets), including this cover page and one scratch page at the end.
- Allow 50 minutes.
- Work as though this were a closed-book exam. You may use one $8\frac{1}{2}'' \times 11''$ sheet of handwritten notes (front and back) and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, give answers in exact form (for example, $\pi/4$ or $\sqrt{2}$ rather than decimals).
- In order to receive full credit, show all your work.
- **Write your answers in the provided blanks or clearly box your final answers.**
- If you need more room, use the scratch page at the end and clearly indicate where your additional work can be found.
- **Do not write within 1 centimeter of the edge of the page.**

Problem	1	2	3	4	5	6	7	Total
Points	6	4	4	12	10	10	14	60

GOOD LUCK!

1. (6 pts) Partial derivatives – *Section: 14.3*

Source: Loveless, Winter 2024 Exam 2, Problem 1(b). Similar practice: HW 14.3; Week 6 Tuesday WS.

Find $f_x(x, y)$ and $f_y(x, y)$ for $f(x, y) = y^x + y\sin(x^2y) + \ln x$.

2. (4 pts) Implicit differentiation – *Sections: 14.3, 14.5*

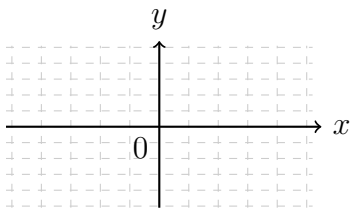
Source: Loveless, Winter 2024 Exam 2, Problem 1(c). Similar practice: HW 14.3; Week 6 Tuesday WS and Thursday TP.

Given $z^3 + y^2z = x\sqrt{3 + y^2}$, find z_y at $(x, y, z) = (1, 1, 1)$ using implicit differentiation.

3. (4 pts) Level curves – *Section: 14.1*

Modeled on: WebAssign HW 14.1, level-curve exercises. Similar practice: HW 14.1 contour/level-curve problems.

For $z = \sqrt{x^2 - y}$, sketch and label the level curves $z = k$ for $k = 0, 1, 2$ on the axes below. Label each curve and mark its y-intercept.



4. (12 pts) Gradients, directions, and the chain rule – *Sections: 14.5, 14.6*

Adapted from: Week 7 Tuesday WS #1 and Week 6 Thursday TP #2. Similar practice: HW 14.5 and HW 14.6.

Let

$$f(x, y) = 3xy - \frac{1}{2}y^2 + 2x^3 + \frac{9}{2}x^2.$$

(a) [4 pts] Find $\nabla f(1, 0)$ and the directional derivative of f at $(1, 0)$ toward $(4, 4)$.

(b) [3 pts] Find the maximum rate of increase of f at $(1, 0)$ and a unit vector giving the direction in which it occurs.

(c) [5 pts] Suppose $z = f(x, y)$ with $x = s^2 + t$ and $y = st$. Use the chain rule to find z_s and z_t at $(s, t) = (1, 2)$.

5. (10 pts) Find and classify – *Section: 14.7*

Source: Loveless, Spring 2023 Exam 2, Problem 2(a). Similar practice: HW 14.7; Week 7 Tuesday Worksheet.

Find and classify all critical points of $f(x, y) = 6x^2 - 3x^2y + y^3 - 3y$. Show the second derivative test. You do not need the function values at the critical points.

6. (10 pts) Lagrange multipliers – *Section: 14.8*

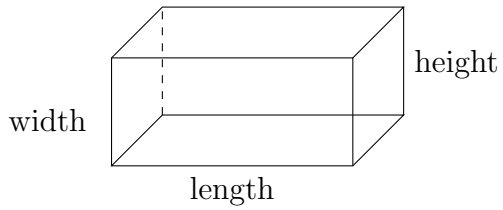
Source: WebAssign HW 14.8, Problem 3 (Stewart 14.8.005). Similar practice: HW 14.8 Problems 2–3.

Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = xy$, subject to $16x^2 + y^2 = 32$.

7. (14 pts) Rectangular mailing package – *Section: 14.8*

Source: WebAssign Optimization Applications HW, Problem 8 (Stewart 14.8.054).

A rectangular mailing box must have length plus girth (the perimeter of its cross-section perpendicular to the length) at most 114 inches. Find the dimensions of the box with the greatest volume. Explain why these dimensions give a maximum. (Length is the longest dimension.)



You may use this page for scratch-work or extra room.

All work on this page will be ignored unless you clearly indicate on the relevant problem that additional work appears here, and label that work with the problem number.