MATH 126 – CHAPTER 14 PRACTICE EXAM

Sample A

Your Name

Your Signature

About this practice exam

These problems are drawn from previous Math 126 exams, assigned WebAssign homework, and course worksheets/test preps. They have been reorganized into a complete practice exam. This is one of five practice exams designed to help you prepare for Exam 2. The questions and difficulty are *not* a prediction of the actual exam. Sources and related homework appear below each problem. Try the exam under timed conditions, then use the solutions to review your work.

Instructions

- CHECK that your practice exam contains 7 problems on 6 pages (3 double-sided sheets), including this cover page and one scratch page at the end.
- Allow 50 minutes.
- Work as though this were a closed-book exam. You may use one $8\frac{1}{2}'' \times 11''$ sheet of handwritten notes (front and back) and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, give answers in exact form (for example, $\pi/4$ or $\sqrt{2}$ rather than decimals).
- In order to receive full credit, show all your work.
- **Write your answers in the provided blanks or clearly box your final answers.**
- If you need more room, use the scratch page at the end and clearly indicate where your additional work can be found.
- **Do not write within 1 centimeter of the edge of the page.**

Problem	1	2	3	4	5	6	7	Total
Points	12	6	6	3	11	10	12	60

GOOD LUCK!

1. (12 pts) Linearization and approximation – *Sections: 14.3, 14.4*

Source: Ostroff, Spring 2019 Math 126 Exam 2, Problem 1(a)–(b). Similar practice: HW 14.4; Week 6 TP #1.

Consider the function $f(x, y) = xy^2 + \sqrt{y - x} - e^{x-2}$.

(a) **[8 pts]** Find the linearization $L(x, y)$ of f at $(2, 3)$.

(b) **[4 pts]** Use your answer from part (a) to find an approximate solution for x to

$$9.61x + \sqrt{3.1 - x} - e^{x-2} = 19.1.$$

2. (6 pts) Implicit partial differentiation – *Sections: 14.3, 14.5*

Source: Week 6 Tuesday Worksheet, Problem 2 (shortened). Similar practice: HW 14.5; Week 6 Thursday TP #4.

The surface S is defined implicitly by $\sqrt{x^2+z} - xy + ze^{y-3} = 2$. Near $P = (2, 3, 5)$, suppose $z = g(x, y)$. Find $\dfrac{\partial z}{\partial x}$ at P. Show your work.

3. (6 pts) Chain rule along a curve – *Section: 14.5*

Source: Loveless, Winter 2012 Math 324 Exam 2, Problem 2(b). Similar practice: HW 14.5 #3–4, #8.

A particle moves along a curve on the surface $z = x^2 + \sin y - xy^2$. Suppose $x(1) = 2$, $y(1) = 0$, $x'(1) = 3$, and $y'(1) = -5$. Find $\dfrac{dz}{dt}$ at $t = 1$.

4. (3 pts) Gradient and steepest increase – *Section: 14.6*

Adapted from: WebAssign HW 14.6, #7 (Stewart 14.6.029.MI). Similar practice: Week 7 Tuesday WS #1.

For $f(x, y) = 7\sin(x + 2y)$, find the maximum rate of increase at $\left(\frac{\pi}{6}, \frac{\pi}{12}\right)$ and a unit vector giving the direction in which it occurs.

5. (11 pts) Find and classify critical points – *Section: 14.7*

Source: Ostroff, Autumn 2023 Math 126 Exam 2, Problem 1. Similar practice: HW 14.7; Week 7 Tuesday WS #2.

Find all critical points of $f(x, y) = 6xy + x^3 - y^2$ and classify each as a local maximum, local minimum, or saddle point. Use the second derivative test.

6. (10 pts) Lagrange multipliers – *Section: 14.8*

Source: WebAssign HW 14.8, #2 (Stewart 14.8.003). Similar practice: HW 14.8 #2–3.

Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x^2 - y^2$ subject to $x^2 + y^2 = 4$.

7. (12 pts) Minimum-cost box – *Sections: 14.7, 14.8*

Source: Loveless, Spring 2023 Math 126 Exam 2, Problem 4. Similar practice: HW 14.7–14.8 Optimization Applications #5–6.

Your job is to design a box with a bottom, four side walls, one dividing wall in the middle, and no top. All walls are made of plywood. The total volume must be 6 cubic feet, and plywood costs 3 per square foot.

Find the minimum cost to produce the box. You *must* give a two-variable cost function and find its critical point. You do not have to perform the second derivative test.

You may use this page for scratch-work or extra room.

All work on this page will be ignored unless you clearly indicate on the relevant problem that additional work appears here, and label that work with the problem number.