Exam 4 Facts

Volumes: $\iint_D f(x,y)\,dA$ = signed volume 'above' the xy-axis, 'below' $f(x,y)$ and inside the region D.

We also saw $\iint_D 1\,dA$ = area of D.

To set up a double integral:

1. Integrand(s). Solve for integrand(s). ($z = f(x,y)$).
2. Draw the region.
   (a) Draw the given xy-equations in the xy-plane (label intersections of curves).
   (b) Draw xy-equations that occur from surface intersections (when the z's are equal).
3. Bounds. Set up the double integral(s) using one of our three methods.
4. Evaluate.

Options for set-up (step 3):

$\iint_D f(x,y)\,dA = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dydx$,	$y = g(x)$ = bottom,	$y = h(x)$ = top
$\iint_D f(x,y)\,dA = \int_c^d \int_{p(y)}^{q(y)} f(x,y)\,dxdy$,	$x = p(y)$ = left,	$x = q(y)$ = right
$\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{w(\theta)}^{v(\theta)} f(r\cos(\theta), r\sin(\theta))\,rdrd\theta$,	$r = w(\theta)$ = inner,	$r = v(\theta)$ = outer

Center of Mass Application: If $\rho(x,y)$ = formula for density at a point in the region D, then

$$M = \text{total mass} = \iint_D \rho(x,y)\ dA \quad , \quad \overline{x} = \frac{\iint_D x\rho(x,y)\ dA}{\iint_D \rho(x,y)\ dA} \quad \text{and} \quad \overline{y} = \frac{\iint_D y\rho(x,y)\ dA}{\iint_D \rho(x,y)\ dA}$$