1 $f(x) = \ln(\ln(x))$, $b = e$ $\rightarrow$ $f(e) = \ln(\ln(e)) = \ln(1) = 0$

$f'(x) = \frac{1}{\ln(x)x} = (x\ln(x))^{-1}$ $\rightarrow$ $f'(e) = \frac{1}{e\ln(e)} = \frac{1}{e}$

$f''(x) = -(x\ln(x))^{-2}\left(\ln(x) + x\cdot\frac{1}{x}\right)$ $\rightarrow$ $f''(e) = \frac{-(\ln(e)+1)}{(e\ln(e))^2} = -\frac{2}{e^2}$

$$\boxed{\begin{aligned} T_2(x) &= 0 + \frac{1}{e}(x-e) + \frac{1}{2!}\,\frac{-2}{e^2}(x-e)^2 \\ T_2(x) &= \frac{1}{e}(x-e) - \frac{1}{e^2}(x-e)^2 \end{aligned}}$$

2 $f(x) = \sin\left(\frac{\pi x}{6}\right)$ $b = 1$ $\rightarrow$ $f(1) = \sin(\pi/6) = 1/2$

$f'(x) = \pi/6 \cos(\pi x/6)$ $\rightarrow$ $f'(1) = \pi/6 \cos(\pi/6) = \frac{\pi\sqrt{3}}{12}$

$f''(x) = -\pi^2/6^2 \sin(\pi x/6)$ $\rightarrow$ $f''(1) = -\pi^2/36 \;\; \frac{1}{2} = -\frac{\pi^2}{72}$

(a) $$\boxed{\begin{aligned} T_2(x) &= \frac{1}{2} + \frac{\pi\sqrt{3}}{12}(x-1) - \frac{1}{2!}\frac{\pi^2}{72}(x-1)^2 \\ T_2(x) &= \frac{1}{2} + \frac{\pi\sqrt{3}}{12}(x-1) - \frac{\pi^2}{144}(x-1)^2 \end{aligned}}$$

(b) Interval: $I = [1, 1.1]$

STEP 1 Next Derivative: $f'''(x) = -\frac{\pi^3}{6^3}\cos(\pi x/6)$

STEP 2 MAXIMIZE: $|f'''(x)| = \frac{\pi^3}{6^3}\cos(\pi x/6)$ $\leftarrow$ decreasing on I

$m = \frac{\pi^3}{6^3}\cos\left(\frac{\pi}{6}\right) = \frac{\pi^3}{6^3}\,\frac{\sqrt{3}}{2}$ $\leftarrow$ for a less precise bound you could use 1 here which would still get full credit.

≈ 0.1243158485

STEP 3 TAYLOR'S INEQ.

$$|f(x) - T_2(x)| \le \frac{m}{3!}|\overset{1.1}{x} - 1|^3 \le \frac{m}{6}(0.1)^3$$

$$\approx \boxed{0.0000207193}$$

3 $f(x) = x\ln(x)$ $b = 1$ $\rightarrow$ $f(1) = 1\ln(1) = 0$

$f'(x) = \ln(x) + x\frac{1}{x} = \ln(x) + 1$ $\rightarrow$ $f'(1) = \ln(1) + 1 = 1$

$f''(x) = \frac{1}{x}$ $\rightarrow$ $f''(1) = 1/1 = 1$

(a) $$\boxed{\begin{aligned} T_2(x) &= 0 + 1(x-1) + \frac{1}{2!}1(x-1)^2 \\ T_2(x) &= (x-1) + \frac{1}{2}(x-1)^2 \end{aligned}}$$

(b) $0.9\ln(0.9) \approx (0.9-1) + \frac{1}{2}(0.9-1)^2$

$= -0.1 + \frac{1}{2}0.01 = \boxed{-0.095}$

(c) Interval: $I = [0.9, 1]$

STEP 1 Next Derivative: $f'''(x) = -\frac{1}{x^2}$

STEP 2 Maximize: $|f'''(x)| = \frac{1}{x^2}$ ⇐ decreasing on I

$M = \frac{1}{0.9^2} = \frac{1}{0.81} \approx 1.234567901$

STEP 3 Taylor's Inequality

$|f(x) - T_2(x)| \le \frac{M}{3!}|x-1|^3 \le \frac{M}{6}(0.1)^3 \approx \boxed{0.00020576}$

4 $f(x) = x^3 + x$, $b = 1$ → $f(1) = 2$

$f'(x) = 3x^2 + 1$ $\quad f'(1) = 4$

$f''(x) = 6x$ $\quad f''(1) = 6$

(a) $T_2(x) = 2 + 4(x-1) + \frac{1}{2!}6(x-1)^2$

$T_2(x) = 2 + 4(x-1) + 3(x-1)^2$

(b) ERROR BOUND

$f'''(x) = 6$ for all x in any interval

so $M = 6$

Taylor's Inequality tells us

$|T_2(x) - f(x)| \le \frac{M}{3!}|x-1|^3 = \frac{6}{3!}|x-1|^3 = |x-1|^3$

We want to know which value of x will give an error bound of less than 0.001

$|x-1|^3 < 0.001$, taking the cube root gives

$\Rightarrow |x-1| < 0.1$, which means

$-0.1 < x-1 < 0.1$, and adding 1 gives

$0.9 < x < 1.1$

The interval $\boxed{J = (0.9, 1.1)}$ gives an error bound less than 0.001

5 $\sin(x^2) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}(x^2)^{2k+1} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{4k+2}$ for all x

Thus, $\int_0^2 \sin(x^2)\,dx = \int_0^2 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{4k+2}\,dx$

ACTUAL VALUE 0.8047764893

$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \frac{1}{4k+3} x^{4k+3} \Big|_0^2$

$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \frac{1}{(4k+3)} 2^{4k+3}$

$\approx \frac{1}{1!}\frac{1}{3}2^3 - \frac{1}{3!}\frac{1}{7}2^7 + \frac{1}{5!}\frac{1}{11}2^{11} - \frac{1}{7!}\frac{1}{15}2^{15}$

$= \boxed{0.737123617}$

6 $\dfrac{1}{1+5x} = \sum_{n=0}^{\infty} (-5x)^n = \sum_{n=0}^{\infty} (-1)^n 5^n x^n$ for $-1 < -5x < 1$

$\frac{1}{5} > x > -\frac{1}{5}$

$\dfrac{1}{3+x} = \dfrac{1}{3} \dfrac{1}{1+x/3} = \dfrac{1}{3} \sum_{n=0}^{\infty} \left(-\dfrac{x}{3}\right)^n$ for $-1 < -x/3 < 1$

$= \dfrac{1}{3} \sum_{n=0}^{\infty} \dfrac{(-1)^n}{3^n} x^n$ $\quad 3 > x > -3$

TOGETHER:

$$\frac{1}{1+5x} + \frac{1}{3+x} = \sum_{n=0}^{\infty} \left[(-1)^n 5^n + \frac{(-1)^n}{3^{n+1}}\right] x^n \text{ for } -\tfrac{1}{5} < x < \tfrac{1}{5}$$

FIRST FOUR TERMS:

$$\boxed{\left(1+\tfrac{1}{3}\right) - \left(5+\tfrac{1}{9}\right)x + \left(5^2+\tfrac{1}{27}\right)x^2 - \left(5^3+\tfrac{1}{81}\right)x^3}$$

7 $e^{x^2} = \sum_{n=0}^{\infty} \dfrac{1}{n!}(x^2)^n = \sum_{n=0}^{\infty} \dfrac{1}{n!} x^{2n}$ for all x.

so

$$x^3 e^{x^2} = \sum_{n=0}^{\infty} \frac{1}{n!} x^{2n+3} = x^3 + \frac{1}{1!}x^5 + \frac{1}{2!}x^7 + \frac{1}{3!}x^9 + \frac{1}{4!}x^{11} + \dots$$

$$\frac{1}{4!} = \boxed{\frac{1}{24} = \text{the coefficient of } x^{11}}$$

8 $\cos(5x^2) = \sum_{n=0}^{\infty} \dfrac{(-1)^n}{(2n)!}(5x^2)^{2n} = \sum_{n=0}^{\infty} \dfrac{(-1)^n}{(2n)!} 5^{2n} x^{4n}$ for all x

so

$x^3 \cos(5x^2) = \sum_{n=0}^{\infty} \dfrac{(-1)^n}{(2n)!} 5^{2n} x^{4n+3}$ for all x

$$\int x^3 \cos(5x^2) = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} 5^{2n} x^{4n+3}\, dx$$

$$= \boxed{C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} 5^{2n} \frac{1}{(4n+4)} x^{4n+4}}$$

9 $f(x) = \ln(3+2x^2)$

(a) $f'(x) = \dfrac{4x}{3+2x^2} = \dfrac{4x}{3} \dfrac{1}{1+\frac{2}{3}x^2}$ for

$\dfrac{1}{1+\frac{2}{3}x^2} = \sum_{n=0}^{\infty}\left(-\dfrac{2}{3}x^2\right)^n = \sum_{n=0}^{\infty} (-1)^n \dfrac{2^n}{3^n} x^{2n}$ $\quad -1 < \frac{2}{3}x^2 < 1$

$-\frac{3}{2} < x^2 < \frac{3}{2}$

so

$$\boxed{\frac{4x}{3(1+\frac{2}{3}x^2)} = \sum_{n=0}^{\infty} \frac{4(-1)^n 2^n}{3^{n+1}} x^{2n+1}}$$

$-\sqrt{\frac{3}{2}} < x < \sqrt{\frac{3}{2}}$

(b) $f(x) = \int \frac{4x}{3(1+\frac{2}{3}x^2)}dx = C + \sum_{n=0}^{\infty} \frac{4(-1)^n 2^n}{3^{n+1}(2n+2)} x^{2n+2} = \ln(3+2x^2)$

$x=0 \Rightarrow f(0) = \ln(3) \Rightarrow C = \ln(3)$

Thus, $\boxed{\ln(3+2x^2) = \ln(3) + \sum_{n=0}^{\infty} \frac{4(-1)^n 2^n}{3^{n+1}(2n+2)} x^{2n+2}}$

(c) Interval: $\left(-\sqrt{3/2}, \sqrt{3/2}\right)$ $\boxed{\text{Radius} = \sqrt{3/2}}$

(d) $T_4(x) = \ln(3) + \frac{4}{6}x^2 - \frac{\cancel{4}\cdot 2}{9\cdot\cancel{4}}x^4 = \ln(3) + \frac{2}{3}x^2 - \frac{2}{9}x^4$

Interval: $(-a, a)$

$f'(x) = \frac{4x}{3+2x^2}$

$f''(x) = \frac{(3+2x^2)4 - 4x\cdot 4x}{(3+2x^2)^2} = \frac{12-8x^2}{(3+2x^2)^2}$

$f'''(x) = \frac{(3+2x^2)^2(-16x) - (12-8x^2)2(3+2x^2)4x}{(3+2x^2)^4}$

TOO MESSY TO BE AN EXAM PROBLEM,
THIS WAS INCLUDED IN THE WORKSHEET BY MY ERROR
(THIS QUESTION ACTUALLY WENT WITH A DIFFERENT FUNCTION)
TO DO THIS YOU WOULD NEED TO COMPUTE $f^{(5)}(x)$.

$\boxed{10}$ $\vec{v} = \langle a,b,c \rangle$

Want (i) $\vec{v}$ is orthogonal to $\langle 2,1,4\rangle \Rightarrow \langle a,b,c\rangle \cdot \langle 2,1,4\rangle = 0$

So $\boxed{2a+b+4c=0}$

(ii) $\vec{v} \times \langle 1,2,0\rangle = \langle 2,-1,0\rangle \Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ 1 & 2 & 0 \end{vmatrix} = \langle 2,-1,0\rangle$

$\langle \overset{\mathrm{I}}{b\cdot 0 - 2\cdot c}, \overset{\mathrm{II}}{1\cdot c - a\cdot 0}, \overset{\mathrm{III}}{2\cdot a - 1\cdot b}\rangle = \langle 2,-1,0\rangle$

So $\boxed{\mathrm{I}}$ $-2c = 2 \Rightarrow \boxed{c=-1}$

$\boxed{\mathrm{II}}$ $\boxed{c=-1}$

$\boxed{\mathrm{III}}$ $\boxed{2a-b=0} \Rightarrow 2a = b$

(i) & (ii) give $2a+b+4c=0$

$2a+2a-4=0 \Rightarrow 4a=4$ $\boxed{a=1}$

$b=2a \Rightarrow \boxed{b=2}$

$\boxed{\vec{v} = \langle 1,2,-1\rangle}$

11 (a) Equating components: (i) $2t = 2-2u \Rightarrow t = 1-u$

(ii) $0 = 3u \Rightarrow \boxed{u=0}$ so $\boxed{t=1}$

(iii) $4-4t=0$ ✓

$t=1 \Rightarrow \boxed{(x,y,z) = (2,0,0)}$ ✓

$u=0 \Rightarrow (x,y,z) = (2,0,0)$

(b) Find two vectors parallel to the plane

(or find 3 pts: $P(0,0,4)$, $Q(2,0,0)$, $R(0,3,0)$)

Vectors: $\vec{v}_1 = \langle 2,0,-4\rangle$, $\vec{v}_2 = \langle -2,3,0\rangle$

$\vec{n} = \langle 2,0,-4\rangle \times \langle -2,3,0\rangle = \langle 0--12, 8-0, 6-0\rangle$

$\vec{n} = \langle 12,8,6\rangle$ ✓

$\langle 12,8,6\rangle \cdot \langle x-2, y, z\rangle = 0$

$12(x-2)+8y+6z=0 \Rightarrow \boxed{12x+8y+6z=24}$

12 Find two points of intersection

(or find one point and cross the normals to get direction)

$z=0 \Rightarrow \begin{matrix} x+y=1 \\ 3x-y=1 \end{matrix} \Rightarrow 4x=2 \Rightarrow x=\frac{1}{2}$ $\quad y=\frac{1}{2}$

$(\frac{1}{2}, \frac{1}{2}, 0)$

$y=0 \Rightarrow \begin{matrix} x+2z=1 \\ 3x+4z=1 \end{matrix} \Rightarrow \begin{matrix} 2x+4z=2 \\ 3x+4z=1 \end{matrix} \Rightarrow -x=1 \quad x=-1 \Rightarrow z=1$

$(-1,0,1)$

DIRECTION VECTOR: $\vec{v} = \langle \frac{1}{2}--1, \frac{1}{2}-0, 0-1\rangle = \langle \frac{3}{2}, \frac{1}{2}, -1\rangle$

(or any nonzero multiple of this vector)

say $\vec{v} = \langle 3,1,-2\rangle$ for simplicity.

$\vec{r}_0 = \langle -1,0,1\rangle$ (or any point of intersection)

$\langle x,y,z\rangle = \langle -1,0,1\rangle + t\langle 3,1,-2\rangle$

$\boxed{x=-1+3t,\ y=t,\ z=1-2t}$

13 Through origin $\vec{r}_0 = \langle 0,0,0\rangle$

$\vec{n} = \langle a,b,c\rangle$ is perpendicular to both $\langle 5,-1,1\rangle$ and $\langle 3,2,-3\rangle$

$\langle 5,-1,1\rangle \times \langle 3,2,-3\rangle = \langle 3-2, 3--15, 10--3\rangle = \langle 1,18,13\rangle$

$\langle 1,18,13\rangle \cdot \langle x-0,y-0,z-0\rangle = 0$

$\boxed{x+18y+13z=0}$

14

$$\vec{r}(t) = \sin(t)\,\vec{i} + \cos(t)\,\vec{j} + t\,\vec{k}$$

$$\vec{r}\,'(t) = \cos(t)\,\vec{i} - \sin(t)\,\vec{j} + \vec{k} \qquad \vec{r}\,'(1) = \cos(1)\,\vec{i} - \sin(1)\,\vec{j} + \vec{k}$$

$$\vec{r}\,''(t) = -\sin(t)\,\vec{i} - \cos(t)\,\vec{j} \qquad \vec{r}\,''(1) = -\sin(1)\,\vec{i} - \cos(1)\,\vec{j}$$

$$a_n = \frac{|\vec{r}\,'(1) \times \vec{r}\,''(1)|}{|\vec{r}\,'(1)|} = \frac{|\langle 0 - -\cos(1), -\sin(1) - 0, \overbrace{-\cos^2(1) - \sin^2(1)}^{=-1}\rangle|}{|\langle \cos(1), -\sin(1), 1\rangle|}$$

$$= \sqrt{\frac{\cos^2(1) + \sin^2(1) + 1}{\cos^2(1) + \sin^2(1) + 1}} = \boxed{1}$$

15 (a) $\vec{r}(t) = \langle \cos(t), \cos(t), \sqrt{2}\sin(t)\rangle$

$$\boxed{\vec{r}\,'(t) = \langle -\sin(t), -\sin(t), \sqrt{2}\cos(t)\rangle}$$

(b)

$$s(t) = \int_0^t |\vec{r}\,'(u)|\,du = \int_0^t \sqrt{\overbrace{\sin^2(u) + \sin^2(u) + 2\cos^2(u)}^{2(\sin^2(u)+\cos^2(u))=2}}\,du$$

$$\Rightarrow \quad s(t) = \int_0^t \sqrt{2}\,du = \sqrt{2}\,u\Big|_0^t = \sqrt{2}\,t$$

$$s = \sqrt{2}\,t \quad \Rightarrow \quad \boxed{t = \frac{1}{\sqrt{2}}\,s = \frac{\sqrt{2}}{2}\,s}$$

$$\boxed{\vec{r}(t(s)) = \left\langle \cos\left(\tfrac{\sqrt{2}}{2}s\right), \cos\left(\tfrac{\sqrt{2}}{2}s\right), \sqrt{2}\sin\left(\tfrac{\sqrt{2}}{2}s\right)\right\rangle}$$

(c) $\left(\frac{1}{2}, \frac{1}{2}, \sqrt{\frac{3}{2}}\right) = (\cos(t), \cos(t), \sqrt{2}\sin(t)) \Rightarrow t = \pi/3$

(i) TANGENT LINE: direction $\vec{v} = \vec{r}\,'(\pi/3) = \langle -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}\rangle$

$$x = \tfrac{1}{2} - \tfrac{\sqrt{3}}{2}t, \quad y = \tfrac{1}{2} - \tfrac{\sqrt{3}}{2}t, \quad z = \sqrt{\tfrac{3}{2}} + \tfrac{\sqrt{2}}{2}t$$

(ii) CURVATURE:

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle -\sin(t), -\sin(t), \sqrt{2}\cos(t)\rangle}{\sqrt{\sin^2(t) + \sin^2(t) + 2\cos^2(t)}}$$

$$= \left\langle -\tfrac{\sqrt{2}}{2}\sin(t), -\tfrac{\sqrt{2}}{2}\sin(t), \cos(t)\right\rangle$$

$$\vec{T}(s) = \left\langle -\tfrac{\sqrt{2}}{2}\sin\left(\tfrac{\sqrt{2}}{2}s\right), -\tfrac{\sqrt{2}}{2}\sin\left(\tfrac{\sqrt{2}}{2}s\right), \cos\left(\tfrac{\sqrt{2}}{2}s\right)\right\rangle$$

$$\kappa = \left|\frac{d\vec{T}}{ds}\right| = \left|\left\langle -\tfrac{1}{2}\cos\left(\tfrac{\sqrt{2}}{2}s\right), -\tfrac{1}{2}\cos\left(\tfrac{\sqrt{2}}{2}s\right), -\tfrac{\sqrt{2}}{2}\sin\left(\tfrac{\sqrt{2}}{2}s\right)\right\rangle\right|$$

$$= \sqrt{\tfrac{1}{4}\cos^2\left(\tfrac{\sqrt{2}}{2}s\right) + \tfrac{1}{4}\cos^2\left(\tfrac{\sqrt{2}}{2}s\right) + \tfrac{1}{2}\sin^2\left(\tfrac{\sqrt{2}}{2}s\right)}$$

$$\boxed{\kappa = \sqrt{\tfrac{1}{2}} = \tfrac{\sqrt{2}}{2}}$$

(iii)

$$\vec{N}(t) = \frac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|} = \frac{\langle -\frac{\sqrt{2}}{2}\cos(t), -\frac{\sqrt{2}}{2}\cos(t), -\sin(t)\rangle}{\underbrace{\sqrt{\frac{1}{2}\cos^2(t) + \frac{1}{2}\cos^2(t) + \sin^2(t)}}_{=1}}$$

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t) = \left\langle \tfrac{\sqrt{2}}{2}\sin^2(t) + \tfrac{\sqrt{2}}{2}\cos^2(t), -\tfrac{\sqrt{2}}{2}\cos^2(t) - \tfrac{\sqrt{2}}{2}\sin^2(t), 0\right\rangle$$

$$= \left\langle \tfrac{\sqrt{2}}{2}, -\tfrac{\sqrt{2}}{2}, 0\right\rangle$$

SKIP

OSCULATING PLANE: $\vec{n} = \vec{B}(\pi/3) = \langle \sqrt{2}/2, -\sqrt{2}/2, 0 \rangle$

$\langle \sqrt{2}/2, -\sqrt{2}/2, 0\rangle \cdot \langle x-1/2, y-1/2, z-\sqrt{2}/2\rangle = 0$

$\sqrt{2}/2 (x-1/2) - \sqrt{2}/2 (y-1/2) = 0$

$x - 1/2 - (y - 1/2) = 0$ $\quad \boxed{x-y=0}$

SKIP

(iv) NORMAL PLANE: $\vec{n} = \vec{r}'(\pi/3) = \langle -\sqrt{3}/2, -\sqrt{2}/2, \sqrt{2}/2 \rangle$

$\langle -\sqrt{3}/2, -\sqrt{2}/2, \sqrt{2}/2\rangle \cdot \langle x-1/2, y-1/2, z-\sqrt{2}/2\rangle = 0$

$\boxed{-\frac{\sqrt{3}}{2}(x-1/2) - \sqrt{2}/2 (y-1/2) + \sqrt{2}/2 (z - \sqrt{2}/2) = 0}$

$\boxed{16}$ $\vec{r}(t) = \langle 3+t, 2+\ln(t), 7+t^2 \rangle$ $\quad \vec{r}'(t) = \langle 1, 1/t, 2t \rangle$ can't use t here already in use

Tangent Line: $\langle x, y, z\rangle = \langle 3+t, 2+\ln(t), 7+t^2\rangle + u\langle 1, 1/t, 2t\rangle$

What value of t will make it so the line goes through $(7,5,14)$?

(i) $7 = 3+t+u \Rightarrow 4 = t+u \Rightarrow u = 4-t$

(ii) $5 = 2+\ln(t) + u/t$

(iii) $14 = 7+t^2+2ut$

(i) & (iii) $\Rightarrow$ $14 = 7+t^2+2(4-t)t$

$7 = t^2 + 8t - 2t^2$

$t^2 - 8t + 7 = 0$ $\quad (t-1)(t-7) = 0$

$t = 1$ or $t = 7$

$u = 4-1 = 3$ $\quad u = 4-7 = -3$

check (ii) $5 = 2 + \ln(t) + u/t$

$3 = \ln(t) + u/t$

$t=1, u=3$ work ✓

$t=7, u=-3$ does not

$\boxed{t=1}$

$\boxed{17}$ $\vec{v}(t) = \int \vec{a}(t)\,dt = \int \vec{i} - 12t^2\vec{j} + 2t\vec{k}\,dt = (t+c_1)\vec{i} + (-4t^3+c_2)\vec{j} + (t^2+c_3)\vec{k}$

$\vec{v}(0) = 2\vec{j} \Rightarrow c_1 = 0, c_2 = 2, c_3 = 0$

$\vec{v}(t) = t\vec{i} + (-4t^3+2)\vec{j} + t^2\vec{k}$

$\vec{r}(t) = \int \vec{v}(t)\,dt = (\frac{1}{2}t^2 + d_1)\vec{i} + (-t^4 + 2t + d_2)\vec{j} + (\frac{1}{3}t^3 + d_3)\vec{k}$

$\vec{r}(0) = \vec{i} + \vec{k} \Rightarrow d_1 = 1, d_2 = 0, d_3 = 1$

$\boxed{\vec{r}(t) = (\frac{1}{2}t^2+1)\vec{i} + (-t^4+2t)\vec{j} + (\frac{1}{3}t^3+1)\vec{k}}$

18 $f(x,y) = e^{3x+5y-1}$

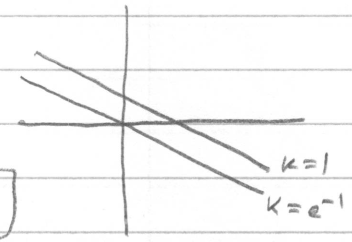
(a) $k = e^{-1} \Rightarrow e^{-1} = e^{3x+5y-1} \Rightarrow -1 = 3x+5y-1$

$0 = 3x+5y \qquad y = -\frac{3}{5}x$

LINES

$k = 1 \Rightarrow 1 = e^{3x+5y-1} \Rightarrow 3x+5y-1 = 0 \qquad y = -\frac{3}{5}x + \frac{1}{5}$

There are no points corresponding to $k \leq 0$.



(b) $\boxed{f_x(x,y) = 3e^{3x+5y-1}}$ $\boxed{f_y(x,y) = 5e^{3x+5y-1}}$

(c) $f_x(2,-1) = 3e^{6-5-1} = 3e^{\cancel{-7}\,0} = 3$

$f_y(2,-1) = 5e^{\cancel{-7}\,0} = 5$

TANGENT PLANE: $\boxed{z-1 = 3\cancel{e^{-7}}(x-2) + 5\cancel{e^{-7}}(y--1)}$

(d) $L(x,y) = 1 + 3\cancel{e^{-7}}(x-2) + 5\cancel{e^{-7}}(y+1)$

$f(1.8,-0.9) \approx 1 + 3\cancel{e^{-7}}(1.8-2) + 5\cancel{e^{-7}}(-0.9+1)$

$= 1 + 3\cancel{e^{-7}}(-0.2) + 5\cancel{e^{-7}}(0.1)$

$= 1 - 0.6\cancel{e^{-7}} + 0.5\cancel{e^{-7}} = \boxed{1 - 0.1\cancel{e^{-7}}}$

19 $f(x,y) = x^3 + y^2 + 2xy$

(i) $f_x(x,y) = 3x^2 + 2y \overset{?}{=} 0$

(ii) $f_y(x,y) = 2y + 2x = 0 \Longrightarrow y = -x$

(i) & (ii) $3x^2 + 2(-x) = 0 \Rightarrow x(3x-2) = 0$

$x = 0$ or $3x - 2 = 0 \qquad x = \frac{2}{3}$

$y = 0 \qquad \boxed{(0,0) \qquad (\tfrac{2}{3}, -\tfrac{2}{3})} \qquad y = -\frac{2}{3}$

$f_{xx} = 6x, \quad f_{yy} = 2, \quad f_{xy} = 2, \quad D = 12x - 4$

$(0,0) \Rightarrow D(0,0) = -4 < 0$ SADDLE POINT

$(\frac{2}{3}, -\frac{2}{3}) \Rightarrow D(\frac{2}{3}, -\frac{2}{3}) = 8 - 4 = 4 > 0$ LOCAL MIN

$f_{xx}(\frac{2}{3}, -\frac{2}{3}) = 4 > 0$

20 $x+y=2 \Rightarrow y=-x+2$

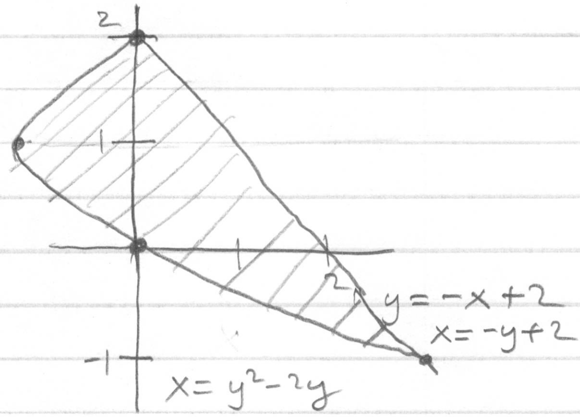
$y^2-2y-x=0 \Rightarrow y^2-2y=x$

$y(y-2)=x$

intersect $y^2-2y=2-y$

$y^2-y-2=0$

$(y-2)(y+1)=0$



$\iint_D x+y \, dA$

$-1 \le y \le 2$

$y^2-2y \le x \le -y+2$

$\int_{-1}^{2} \int_{y^2-2y}^{-y+2} x+y \, dx \, dy$

$= \int_{-1}^{2} \frac{1}{2}x^2+yx \Big|_{y^2-2y}^{-y+2} dy = \int_{-1}^{2} \left[\left(\frac{1}{2}(-y+2)^2+y(-y+2)\right) - \left(\frac{1}{2}(y^2-2y)^2+y(y^2-2y)\right)\right] dy$

$= \int_{-1}^{2} \frac{1}{2}(y^2-4y+4)-y^2+2y-\frac{1}{2}(y^4-4y^3+4y^2)-y^3+2y^2 \, dy$

$= \int_{-1}^{2} \frac{1}{2}y^2-\cancel{2y}+2-y^2+\cancel{2y}-\frac{1}{2}y^4+2y^3-\cancel{2y^2}-y^3+\cancel{2y^2} \, dy$

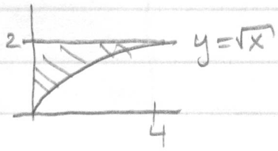
$= \int_{-1}^{2} -\frac{1}{2}y^4+y^3-\frac{1}{2}y^2+2 \, dy = -\frac{1}{10}y^5+\frac{1}{4}y^4-\frac{1}{6}y^3+2y \Big|_{-1}^{2}$

$= \left(-\frac{1}{10}2^5+\frac{1}{4}2^4-\frac{1}{6}2^3+2(2)\right)-\left(-\frac{1}{10}(-1)^5+\frac{1}{4}(-1)^4-\frac{1}{6}(-1)^3+2(-1)\right)$

$= \boxed{\frac{99}{20} \doteq 4.95}$

21 (a) $0 \le x \le 4$

$\sqrt{x} \le y \le 2$



$0 \le y \le 2$

$0 \le x \le y^2$

$\int_0^2 \int_0^{y^2} xy \, dx \, dy$

(b) $\int_0^2 \int_0^{y^2} xy \, dx \, dy = \int_0^2 \frac{1}{2}x^2 y \Big|_0^{y^2} dy = \frac{1}{2}\int_0^2 y^5 \, dy$

$= \frac{1}{12}y^6 \Big|_0^2 = \frac{2^6}{12} = \boxed{\frac{16}{3}}$

$\int_0^4 \int_{\sqrt{x}}^{2} xy \, dy \, dx = \int_0^4 \frac{1}{2}xy^2 \Big|_{\sqrt{x}}^{2} dx = \int_0^4 2x-\frac{1}{2}x^2 \, dx$

$= x^2-\frac{1}{6}x^3 \Big|_0^4 = 4^2-\frac{1}{6}4^3 = 16\left(1-\frac{4}{6}\right) = \boxed{\frac{16}{3}}$

22 $\int_0^3 \int_0^{9-x^2} \frac{xe^{3y}}{9-y}\, dy\, dx$

CAN'T INTEGRATE, NEED TO REVERSE ORDER

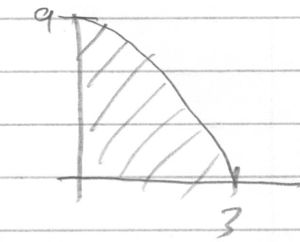
$0 \le x \le 3$

$0 \le y \le 9 - x^2$

$y = 9 - x^2$

$x^2 = 9 - y$

$x = \pm\sqrt{9-y}$



$0 \le y \le 9$

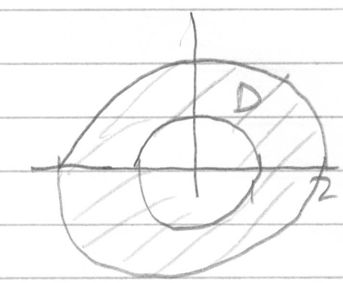
$0 \le x \le \sqrt{9-y}$

$\int_0^9 \int_0^{\sqrt{9-y}} \frac{e^{3y}}{9-y}\, x\, dx\, dy$

$\int_0^9 \frac{e^{3y}}{9-y} \frac{1}{2} x^2 \Big|_0^{\sqrt{9-y}} dy = \int_0^9 \frac{e^{3y}}{\cancel{9-y}} \frac{1}{2} \cancel{(9-y)}\, dy$

$= \frac{1}{6} e^{3y} \Big|_0^9 = \boxed{\frac{1}{6} e^{27} - \frac{1}{6}} = \frac{1}{6}\left(e^{27} - 1\right)$

23 $\iint_D y^2\, dA$



$0 \le \theta \le 2\pi$

$1 \le r \le 2$

$\int_0^{2\pi} \int_1^2 r^2 \sin^2(\theta)\, r\, dr\, d\theta$

$\int_0^{2\pi} \sin^2(\theta) \quad \frac{1}{4} r^4 \Big|_1^2 d\theta$

$\int_0^{2\pi} \sin^2(\theta) \left(\frac{1}{4} 2^4 - \frac{1}{4}\right) d\theta$

$\frac{15}{4} \int_0^{2\pi} \sin^2(\theta)\, d\theta$

$\frac{15}{4} \int_0^{2\pi} \frac{1}{2}(1 - \cos(2\theta))\, d\theta$

$\frac{15}{8} \left[\theta - \frac{1}{2}\sin(2\theta) \Big|_0^{2\pi}\right] = \frac{15}{8}(2\pi) = \boxed{\frac{15\pi}{4}}$