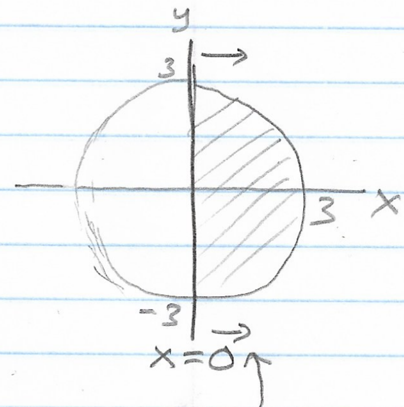
15.3 - LIVE-STREAM

EXAMPLE 1 FIND THE VOLUME OF THE SOLID ABOVE $z=0$, BELOW $z=x$, AND WITHIN $x^2+y^2=9$.

STEP 1 $z=x$ & $z=0$ $\Rightarrow \iint_R x \, dA$



STEP 2 DRAW/LABEL REGION

① GIVEN xy EQUATIONS $\Rightarrow x^2+y^2=9$

② INTERSECTION OF SURFACES?

$z=x$ & $z=0$ $\Rightarrow x=0$

WANT $z>0$, SINCE $z=x$

WANT $x>0$.

STEP 3 SET UP BOUNDS (3 WAYS, ONLY NEED TO DO ONE, YOU PICK):

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} x \, dy \, dx$$

$$\int_{-3}^3 \int_0^{\sqrt{9-y^2}} x \, dx \, dy$$

$$\int_{-\pi/2}^{\pi/2} \int_0^3 r\cos\theta \; r \, dr \, d\theta = \int_{-\pi/2}^{\pi/2} \int_0^3 \cos(\theta) \, r^2 \, dr \, d\theta$$

ALL SHOULD GIVE THIS

STEP 4 EVALUATE

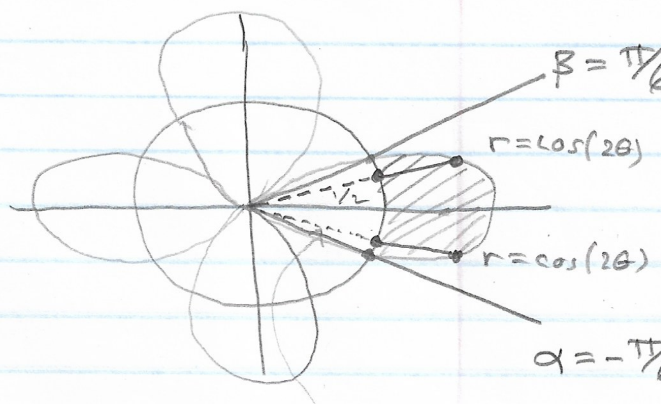
I PICK THIS ONE

$$\int_{-\pi/2}^{\pi/2} \cos(\theta) \, \frac{1}{3} r^3 \Big|_0^3 \, d\theta = \int_{-\pi/2}^{\pi/2} 9\cos\theta \, d\theta$$

$$= 9\sin\theta \Big|_{-\pi/2}^{\pi/2}$$

$$= 9(1) - 9(-1) = \boxed{18}$$

EXAMPLE 2 FIND THE $\overset{\text{SHADED}}{\vee}$ AREA OF THE REGION OUTSIDE THE CIRCLE $x^2+y^2=\frac{1}{4}$ AND INSIDE THE ROSE $r=\cos(2\theta)$. (SHOWN BELOW)



STEP 1 $z=1$

ASIDE

FROM DISCUSSION OTHER DAY, IDEA:

VOLUME = 1 · AREA

$$\iint_R 1\, dA = (\text{AREA OF } R)\cdot 1 = \text{AREA OF } R$$

STEP 2 DRAW/LABEL REGION

$$x^2+y^2 = 1/4 \Rightarrow r^2 = 1/4 \Rightarrow r = 1/2$$

INTERSECTION: $\cos(2\theta) = \frac{1}{2} \Rightarrow 2\theta = \ldots, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$ (UNIT CIRCLE)

$$\Rightarrow \theta = \ldots, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$$

STEP 3 SET-UP BOUNDS (VERY HARD TO DO dxdy OR dydx, USE POLAR)

$$\int_{-\pi/6}^{\pi/6}\int_{\frac{1}{2}}^{\cos(2\theta)} 1\cdot r\,dr\,d\theta \qquad = \underbrace{2\int_0^{\pi/6}}_{\text{ALSO COULD USE SYMMETRY}}\int_{1/2}^{\cos(2\theta)} 1\cdot r\,dr\,d\theta$$

STEP 4 EVALUATE

$$\int_{-\pi/6}^{\pi/6} \frac{1}{2}r^2\Big|_{1/2}^{\cos(2\theta)} d\theta = \int_{-\pi/6}^{\pi/6} \frac{1}{2}\cos^2(2\theta) - \frac{1}{8}\, d\theta$$

$$= \int_{-\pi/6}^{\pi/6} \frac{1}{4}\left(1+\cos(4\theta)\right) - \frac{1}{8}\, d\theta \qquad \frac{1}{4}-\frac{1}{8} = \frac{2}{8}-\frac{1}{8} = \frac{1}{8}$$

$$= \int_{-\pi/6}^{\pi/6} \frac{1}{8} + \frac{1}{4}\cos(4\theta)\, d\theta$$

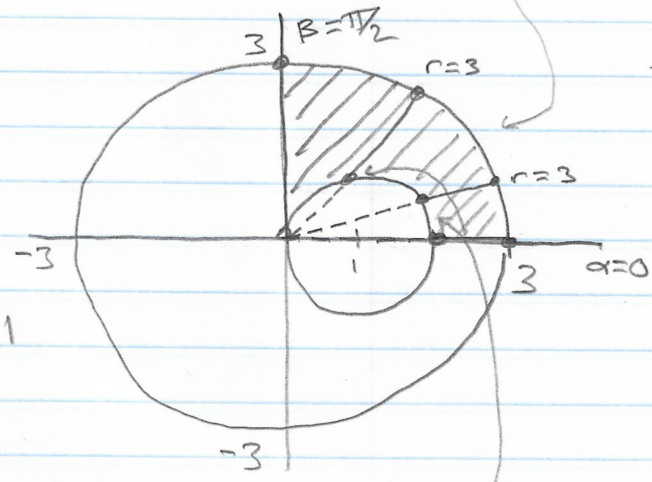
$$= \frac{1}{8}\theta + \frac{1}{16}\sin(4\theta)\Big|_{-\pi/6}^{\pi/6}$$

$$= \left(\frac{\pi}{48} + \frac{1}{16}\overbrace{\sin\left(\frac{2\pi}{3}\right)}^{\sqrt{3}/2}\right) - \left(-\frac{\pi}{48} + \frac{1}{16}\overbrace{\sin\left(-\frac{2\pi}{3}\right)}^{-\sqrt{3}/2}\right)$$

$$= \frac{2\pi}{48} + \frac{2\sqrt{3}}{16\cdot 2} = \boxed{\frac{\pi}{24} + \frac{\sqrt{3}}{16}}$$

$\boxed{\text{EXAMPLE 3}}$ LET R BE THE REGION IN THE <u>FIRST QUADRANT</u> THAT IS INSIDE THE CIRCLE $x^2+y^2=9$, AND OUTSIDE THE CIRCLE $x^2+y^2=2x$.

EVALUATE
$$\iint_R \frac{y}{x^2+y^2}\, dA$$



$\boxed{\text{STEP 1}}$ $z = \frac{y}{x^2+y^2}$ IS GIVEN

$\boxed{\text{STEP 2}}$ DRAW/LABEL REGION

HELPS FOR DRAWING
$$\left\{\begin{array}{l} x^2+y^2=2x \Rightarrow x^2-2x+1+y^2=0+1 \\ \Rightarrow (x-1)^2+y^2=1 \end{array}\right.$$

POLAR:
$$x^2+y^2=9 \Rightarrow r^2=9 \Rightarrow r=3$$
$$x^2+y^2=2x \Rightarrow r^2=2r\cos\theta \underset{r\neq 0}{\Rightarrow} r=2\cos\theta$$

$\boxed{\text{STEP 3}}$ SET UP BOUNDS ($dx\,dy$ WOULD BE AWFUL, $dy\,dx$ WOULD BE BAD, USE POLAR!)

BREAK INTO 3 REGIONS   BREAK INTO 2 REGIONS

$$\int_0^{\pi/2}\int_{2\cos\theta}^{3} \frac{r\sin\theta}{r^2}\, r\, dr\, d\theta$$

$\boxed{\text{STEP 4}}$ EVALUATE

$$\int_0^{\pi/2}\int_{2\cos\theta}^{3} \sin\theta\, dr\, d\theta = \int_0^{\pi/2} \sin(\theta)\, r\Big|_{2\cos\theta}^{3} d\theta$$

$$= \int_0^{\pi/2} 3\sin\theta - 2\sin\theta\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} 3\sin\theta\, d\theta - 2\int_0^{\pi/2}\sin\theta\cos\theta\, d\theta$$

$$= -3\cos\theta\Big|_0^{\pi/2} - 2\int_0^1 u\, du$$

ODD POWER → SUBSTITUTION (OR IDENTITY WORKS HERE)

$u=\sin\theta$
$du=\cos\theta\, d\theta$

$$= (0 - -3) - 2\left(\tfrac{1}{2}u^2\Big|_0^1\right)$$

$$= 3 - (1^2-0^2) = \boxed{2}$$