Math 126 - Spring 2017
Exam 1
April 20, 2017

Name: _____

Section: _____

Student ID Number: _____

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- There are 4 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\frac{7}{2} - \frac{3}{5} = \frac{29}{10}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.
- Show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- If you need more room, use backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board.
  DO NOT CHEAT OR DO ANYTHING THAT LOOKS SUSPICIOUS!
  WE WILL REPORT YOU AND YOU MAY BE EXPELLED!
  Keep your eyes down and on your paper. If your TA sees your eyes wandering they will warn you only once before taking your exam from you.
- You have 50 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 10 MINUTES PER PAGE!

GOOD LUCK!

1. (14 pts) **For ALL parts below**, consider the plane, $\mathcal{P}$, through A(0,0,1), B(1,1,3), and C(-1,2,4).

   (a) To the nearest **degree**, find the angle at A in the triangle BAC. (That is, find $\angle BAC$).

   (b) Find the (x, y, z) point where $\mathcal{P}$ intersects the y-axis. (Hint: Find the equation for the plane).

   (c) A particle starts at the point $(70, 0, 1)$ and moves toward the plane along a straight line that is orthogonal to the plane. At what point, (x, y, z), would this line intersect the plane?

2. (12 pts)

(a) Consider the surface containing all points that satisfy $x^2 + z^2 = 16 + y^2$.

i. Give the 2D names for the traces when the given variable is a constant k:

- For $x = k$, the traces are: _____
- For $y = k$, the traces are: _____

ii. Give the precise 3D name we used in class for this shape:

iii. A particle is traveling **on** the surface $x^2 + z^2 = 16 + y^2$ in such a way that y is negative, $x = 5\cos(t)$ and $z = 5\sin(t)$ and for all times t. Find the value, or formula, for the y-coordinate of the particle at all times t.

(b) Consider the curve given by the polar equation $r = 6\cos(\theta)$.

i. Convert to Cartesian coordinates, then draw a rough sketch of it in the xy-plane with several points labeled. (Hint: It is a shape you know well).

ii. There are two Cartesian points on this graph where the tangent lines are horizontal. Find polar coordinates (r, θ) for these points. (Hint: You can use the graph.).

3. (13 pts) **For ALL parts on this page**, two particles travel along curves given by

$$\mathbf{r}_1(t) = \langle 2t, 3t^2, 2t^3 \rangle \quad \text{and} \quad \mathbf{r}_2(t) = \langle 2 - 2t, 3 + 3t, 2 - 6t \rangle,$$

where t is time in seconds and distances are in feet.

(a) Find a vector that is **tangent** to $\mathbf{r}_1(t)$ at $t = 1$ and has length 10.

(b) Consider $\mathbf{r}_2(t)$ which starts at $(2, 3, 2)$.

i. Find and simplify the arc length function, $s = s(t) = \int_0^t |\mathbf{r}_2'(u)| du$.

ii. Assume the particle stops at the instant it has traveled 28 feet from its starting location. Give its (x, y, z) coordinates at this instant.

(c) Find the (x, y, z) point(s) at which the **paths** of the two particles intersect. (This is NOT a collision question).

4. (11 pts) **NOTE: The two parts below are NOT related!**

(a) Find all values of t at which the tangent line to the curve $x = 8 - t^3$, $y = 42t - 10t^2$ is orthogonal to the vector $\langle 2, -3 \rangle$.

(b) A small bug is moving according to the vector function

$$\mathbf{r}(t) = \langle t \sin(\pi t), \ln(t), t^2 - 4e^{(2-2t)} \rangle.$$

At time $t = 1$ the bug leaves the curve and follows the path of the tangent line. Find the (x, y, z) coordinates where the bug's tangent line path would intersect the xy-plane.