1 (5 points) Calculate the equation of the tangent line to the curve $r = 1 + 2\cos(\theta)$ at the point where $\theta = \pi/2$. Give your equation in terms of x and y.

Point: $r = 1 + x\cos\pi/2 = 1, \quad x = r\cos\theta = 1 \cdot \cos\pi/2 = 0, \quad y = r\sin\theta = 1 \cdot \sin\pi/2 = 1$

Slope: $x = r\cos\theta = (1 + 2\cos\theta)\cos\theta, \quad dx/d\theta = -2\sin\theta\cos\theta - (1 + 2\cos\theta)\sin\theta.$
At $\theta = \pi/2$ *we have* $dx/d\theta = -1.$

$y = r\sin\theta = (1 + 2\cos\theta)\sin\theta, \quad dx/d\theta = -2\sin^2\theta + (1 + 2\cos\theta)\cos\theta.$
At $\theta = \pi/2$ *we have* $dy/d\theta = -2.$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = 2$$

Line: $y - 1 = 2x$

2 (5 points) Compute the distance from the point $(3, 2, 1)$ to the plane $x + 2y + 3z = 1$.

Plug in $y = 0$ *and* $z = 0$ *to get the point* $(1, 0, 0)$ *on the plane.*
(Any point on the plane will do.)

Let $\mathbf{v}$ *be the vector from* $(1, 0, 0)$ *to* $(3, 2, 1)$. *Then* $\mathbf{v} = \langle 2, 2, 1\rangle$.

The distance from $(3, 2, 1)$ *to the plane is the magnitude of the projection of* $\mathbf{v}$ *onto* $\mathbf{N} = \langle 1, 2, 3\rangle$, *the normal vector of the plane.*

$$\textit{distance} = \frac{\mathbf{v} \cdot \mathbf{N}}{|\mathbf{N}|} = \frac{2 \cdot 1 + 2 \cdot 2 + 1 \cdot 3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{9}{\sqrt{14}}.$$

3 (8 points) Compute parametric equations for the line that contains the point $(-1, 2, -3)$ and is parallel to both of the planes $2x - y = 3$ and $x - 2y + 3z = 2$.

The direction vector is $\langle 2, -1, 0\rangle \times \langle 1, -2, 3\rangle = \langle -3, -6, -3\rangle$.

Any nonzero scalar multiple will work so I'll use $\langle 1, 2, 1\rangle$.

The parametric equations are

$$\begin{cases} x &= t - 1 \\ y &= 2t + 2 \\ z &= t - 3 \end{cases}$$

4 (6 points) Find a vector function $\mathbf{r}(t)$ that represents the curve of intersection of the surfaces $4x^2 + (z - 1)^2 = 9$ and $y = 3x^2$.

Rewrite the first equation as $\left(\frac{2x}{3}\right)^2 + \left(\frac{z-1}{3}\right)^2 = 1$.

So set $\frac{2x}{3} = \cos(t)$ *and* $\frac{z-1}{3} = \sin(t)$.

We have $y = 3 \cdot \left[\frac{3}{2}\cos(t)\right]^2$

The vector function is $\mathbf{r}(t) = \left\langle \frac{3}{2}\cos(t), \frac{27}{4}\cos^2(t), 3\sin(t) + 1 \right\rangle$.

5 (12 points) Let $\mathbf{r}(t) = \langle\, t^3, t^2, t^3 - 2t \,\rangle$.

(a) (6 points) Compute the curvature κ at the point $(-1, 1, 1)$.

The point $(-1,1,1)$ *is at* $t = -1$.
$\mathbf{r}'(t) = \langle 3t^2, 2t, 3t^2 - 2\rangle$ *so* $\mathbf{r}'(-1) = \langle 3, -2, 1\rangle$.
$\mathbf{r}''(t) = \langle 6t, 2, 6t\rangle$ *and* $\mathbf{r}''(-1) = \langle -6, 2, -6\rangle$.
Then $\mathbf{r}'(-1) \times \mathbf{r}''(-1) = \langle 10, 12, -6\rangle$ *and*

$$\kappa(-1) = \frac{|\mathbf{r}'(-1) \times \mathbf{r}''(-1)|}{|\mathbf{r}'(-1)|^3} = \frac{\sqrt{280}}{\left(\sqrt{14}\right)^3} = \frac{\sqrt{5}}{7} \approx 0.32.$$

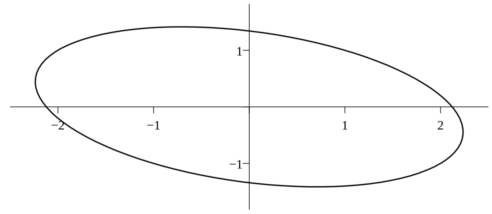
(b) (6 points) Find the arclength of this curve between the points $(-1, 1, 1)$ and $(1, 1, -1)$. Set up the integral, but do not evaluate.

The point $(1, 1, -1)$ *is at* $t = 1$.

We use the arc length formula $s = \int_{-1}^{1} |\mathbf{r}'(t)|\, dt$.

The integral is $\int_{-1}^{1} \sqrt{(3t^2)^2 + (2t)^2 + (3t^2 - 2)^2}\, dt$.

This can be simplified to $\int_{-1}^{1} \sqrt{18t^4 - 8t^2 + 4}\, dt$.



6 (8 points) Find the exact coordinates of the lowest point on the curve in $\mathbf{R}^2$ given by the parametric equations $x = 2\cos(t) + \sin(t)$, $y = \sin(t) - \cos(t)$.

Solve $\dfrac{dy}{dt} = \cos(t) + \sin(t) = 0 \quad \sin(t) = -\cos(t) \quad \tan(t) = -1.$

This means $t = -\dfrac{\pi}{4}$ *or* $\dfrac{3\pi}{4}$.

Note that $\dfrac{dx}{dt} = -2\sin(t) + \cos(t)$ *is non-zero at these values.*

You can tell from the picture that the minimum is at $t = -\frac{\pi}{4}$.

Here $x = 2\cos(-\pi/4) + \sin(-\pi/4) = \dfrac{\sqrt{2}}{2}$ *and* $y = \sin(-\pi/4) - \cos(-\pi/4) = -\sqrt{2}$.

7 (6 points) A particle in $\mathbf{R}^3$ has position function $\mathbf{r}(t) = \langle 2t^3 + 1, t^2, 3t - t^2 \rangle$. Find the speed of the particle when $t = 2$.

$$\begin{aligned} \mathbf{r}'(t) &= \langle 6t^2, 2t, 3 - 2t \rangle \\ \mathbf{r}'(2) &= \langle 24, 4, -1 \rangle \\ |\mathbf{r}'(2)| &= \sqrt{24^2 + 4^2 + (-1)^2} \\ &= \sqrt{593} \end{aligned}$$