1. (14 pts) Evaluate the integrals. In part (c), appropriately change bounds and simplify your final answer.

(a) $\int 7 - e^{-6x} + \frac{5}{2\sqrt[3]{x}}\, dx$ $= \int 7 - e^{-6x} + \frac{5}{2} x^{-1/3}\, dx$

$= 7x + \frac{1}{6} e^{-6x} + \frac{5}{2}\frac{3}{2} x^{2/3} + C$

$= \boxed{7x + \frac{1}{6} e^{-6x} + \frac{15}{4} x^{2/3} + C}$

(b) $\int \frac{\sin(2x)}{\cos(2x)+3}\, dx$

$u = \cos(2x) + 3$

$du = -2\sin(2x)\, dx$

$\frac{1}{-2\sin(2x)}\, du = dx$

$= \int \frac{\cancel{\sin(2x)}}{u} \frac{1}{-2\cancel{\sin(2x)}}\, du$

$= -\frac{1}{2}\ln|u| + C = \boxed{-\frac{1}{2}\ln(\cos(2x)+3) + C}$

(c) $\int_0^1 \frac{\sqrt{1 + \frac{12}{\pi}\tan^{-1}(x)}}{(1+x^2)}\, dx$

$u = 1 + \frac{12}{\pi}\tan^{-1}(x)$

$du = \frac{12}{\pi}\frac{1}{x^2+1}\, dx$

$\frac{\pi}{12}(x^2+1)\, du = dx$

$x = 0 \Rightarrow u = 1 + \frac{12}{\pi}\tan^{-1}(0) = 1$

$x = 1 \Rightarrow u = 1 + \frac{12}{\pi}\tan^{-1}(1)$

$= 1 + \frac{12}{\pi}\frac{\pi}{4} = 1 + 3 = 4$

$\int_1^4 \frac{\sqrt{u}}{\cancel{1+x^2}} \frac{\pi}{12}\cancel{(x^2+1)}\, du$

$= \frac{\pi}{12}\frac{2}{3} u^{3/2} \Big|_1^4$

$= \frac{\pi}{18}\left(4^{3/2} - 1\right) = \boxed{\frac{7\pi}{18}}$

2. (12 pts)

(a) Evaluate: $\displaystyle\int_0^1 \frac{x^7}{(1+x^4)^2}\,dx$

$u = 1 + x^4 \rightarrow x^4 = u - 1$

$du = 4x^3\,dx$

$\frac{1}{4x^3}du = dx$

$= \int_1^2 \frac{x^{\cancel{7}4}}{u^2}\frac{1}{4\cancel{x^3}}du$

$= \frac{1}{4}\int_1^2 \frac{u-1}{u^2}\,du$

$= \frac{1}{4}\int_1^2 \frac{1}{u} - u^{-2}\,du$

$= \frac{1}{4}\left(\ln(u) + \frac{1}{u}\Big|_1^2\right)$

$= \frac{1}{4}\left[\left(\ln(2) + \frac{1}{2}\right) - \left(\ln(1) + 1\right)\right]$

$= \frac{1}{4}\left(\ln(2) - \frac{1}{2}\right) = \frac{1}{4}\ln(2) - \frac{1}{8}$

(b) A table of values for an increasing function f are given:

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	1	3	6	9	12	15	20

i. Approximate the value of $\displaystyle\int_3^5 f(x)\,dx$ using left-endpoints with $n = 4$ subdivisions.

$\Delta x = \frac{5-3}{4} = \frac{1}{2}$ $\quad x_0 = 3,\ x_1 = 3.5,\ x_2 = 4,\ x_3 = 4.5,\ x_4 = 5$

$(1)(\frac{1}{2}) + (3)(\frac{1}{2}) + (6)(\frac{1}{2}) + (9)(\frac{1}{2}) = (1+3+6+9)\frac{1}{2}$

$= \boxed{\frac{19}{2} = 9.5}$

ii. Let $g(x) = \displaystyle\int_3^{x^2+x} f(t)\,dt$. Find the value of the *derivative* of $g(x)$ at $x = 2$. That is, compute $g'(2)$.

$g'(x) = f(x^2+x)\cdot(2x+1)$

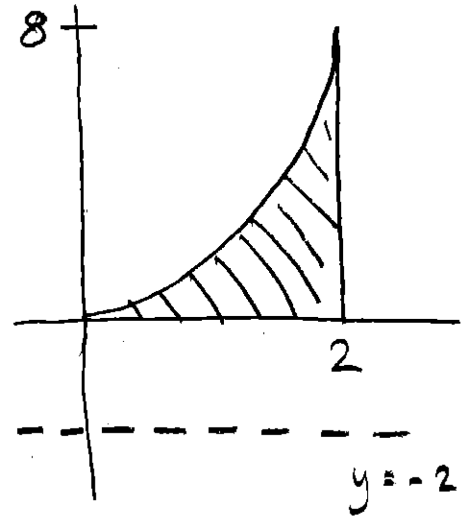
$\Rightarrow g'(2) = f(2^2+2)(2(2)+1) = f(6)\cdot 5$

$= 20\cdot 5 = \boxed{100}$

3. (14 pts)

(a) Let R be the region bounded by $y = x^3$, $x = 2$ and the x-axis. Set up (DO NOT EVALUATE) integrals that represent the volumes of the solids obtained by rotating R about the given axis:

i. ... about the y-axis (any method):



$$\int_0^2 2\pi x \cdot x^3\,dx = \int_0^8 \pi(2)^2 - \pi\left(y^{1/3}\right)^2\,dy$$

BOTH CORRECT

VALUE = $64\pi/5$

ii. ... about the **horizontal** line $y = -2$, using dx:

$$\int_0^2 \pi\left(x^3+2\right)^2 - \pi(2)^2\,dx$$

iii. ... about the **horizontal** line $y = -2$, using dy:

$$\int_0^8 2\pi\,(y+2)\left(2-y^{1/3}\right)dy$$

VALUE = $\frac{240\pi}{7}$

(b) Compute the **area** of the region bounded by $y^2 + x - 2 = 0$ and $x = y$.
(Note: This is a new region, unrelated to the previous question).

INTERSECTIONS $y^2 + y - 2 = 0 \Rightarrow (y+2)(y-1) = 0 \Rightarrow y = -2$ or $y = 1$
$x = -2$ or $x = 1$

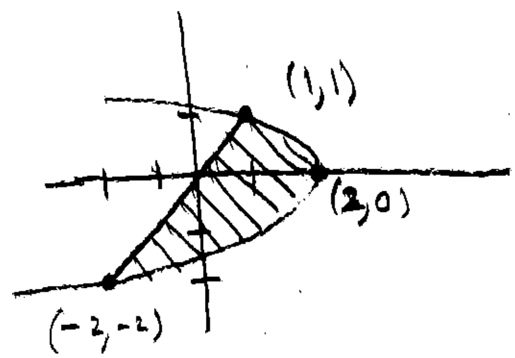
$y^2 + x - 2 = 0 \Rightarrow x = 2 - y^2$

$$\int_{-2}^{1} (2-y^2) - y\,dy$$

$$= 2y - \tfrac{1}{3}y^3 - \tfrac{1}{2}y^2 \Big|_{-2}^{1}$$

$$= \left(2 - \tfrac{1}{3} - \tfrac{1}{2}\right) - \left(-4 + \tfrac{8}{3} - 2\right)$$

$$= 8 - \tfrac{9}{3} - \tfrac{1}{2} = 5 - \tfrac{1}{2} = \boxed{\tfrac{9}{2} = 4.5}$$



ALSO COULD DO

$$\int_{-2}^{1} x - \left(-\sqrt{2-x}\right)dx \leftarrow \tfrac{19}{6}$$

$$+ \int_{1}^{2} \sqrt{2-x} - \left(-\sqrt{2-x}\right)dx \leftarrow \tfrac{4}{3}$$

$$\Big\}\ \tfrac{9}{2}$$

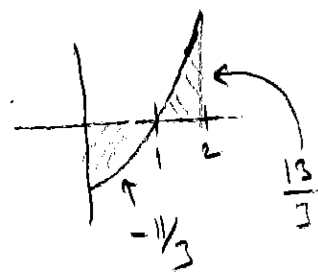
4. (10 pts) The acceleration function (in m/s^2) and the initial velocity, $v(0)$, of a certain particle moving along a line are given by: $a(t) = 2t + 6$ and $v(0) = -7$.
Find the **total distance** traveled by the particle from $t = 0$ to $t = 2$ seconds.
(Hint: First find the velocity function!)

$$a(t) = 2t + 6$$

$$\Rightarrow v(t) = t^2 + 6t + C$$

$$v(0) = -7 \Rightarrow (0)^2 + 6(0) + C = -7 \Rightarrow C = -7$$

$$v(t) = t^2 + 6t - 7$$



WANT

$$\int_0^2 |t^2 + 6t - 7| \, dt$$

① $t^2 + 6t - 7 \stackrel{?}{=} 0 \Rightarrow (t+7)(t-1) \stackrel{?}{=} 0$

$t = -7$ or $t = 1$

② SPLIT!

$$\int_0^1 t^2 + 6t - 7 \, dt = \frac{1}{3}t^3 + 3t^2 - 7t \Big|_0^1 = \frac{1}{3} + 3 - 7$$

$$= \frac{1}{3} - 4 = \frac{1}{3} - \frac{12}{3} = -11/3$$

$$\int_1^2 t^2 + 6t - 7 \, dt = \frac{1}{3}t^3 + 3t^2 - 7t \Big|_1^2$$

$$= \left(\frac{8}{3} + 12 - 14\right) - \left(-11/3\right)$$

$$= \frac{19}{3} - 2 = 13/3$$

TOTAL DISTANCE TRAVELED $= \frac{11}{3} + \frac{13}{3} = \frac{24}{3} = \boxed{8 \text{ meters}}$

5. (10 pts) At time $t = 0$ seconds a small water balloon is **dropped** from the top of a building (so $v(0) = 0$ ft/sec). Dr. Loveless looks up and observes the following:

- At some time, $t = a$ seconds, the balloon passes a window that is 112 feet high.
- One second later, $t = a + 1$ seconds, the balloon hits the ground at his feet.

Assume the balloon fell toward the ground at a constant acceleration of -32 ft/sec^2. How tall is the building?

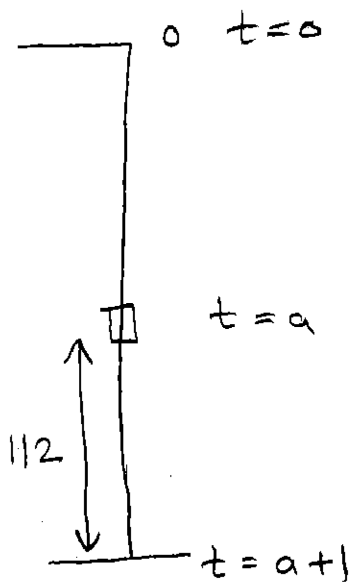
$a(t) = -32$

$v(t) = -32t + C$

$v(0) = 0 \Rightarrow C = 0$

$v(t) = -32t$

$h(t) = -16t^2 + D$



(i) $h(a) = 112 \Rightarrow -16a^2 + D = 112$

(ii) $h(a+1) = 0 \Rightarrow -16(a+1)^2 + D = 0$

(i) $\Rightarrow D = 112 + 16a^2$

(i) & (ii) $\Rightarrow -16(a+1)^2 + 112 + 16a^2 = 0$

$\Rightarrow -16(a^2 + 2a + 1) + 112 + 16a^2 = 0$

$\Rightarrow \cancel{-16a^2} - 32a - 16 + 112 \cancel{+16a^2} = 0$

$96 = 32a$

$\Rightarrow a = \frac{96}{32} = 3$

$\Rightarrow D = 112 + 16(3)^2 = 256$

$\boxed{256 \text{ feet}}$