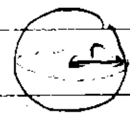
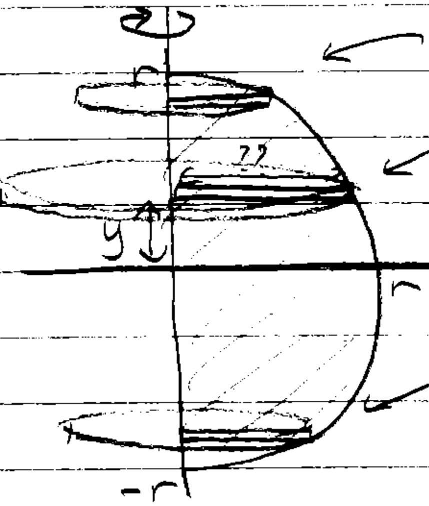
$\boxed{\text{TWO CLASSIC VOLUME PROBLEMS}}$

$\boxed{1}$ $\boxed{\text{SPHERE}}$



CONSIDER THIS REGION
ROTATED ABOUT THE
y-AXIS
IT GIVES A SPHERE!

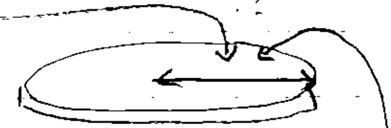


ANY CROSS-SECTION IS A DISC!

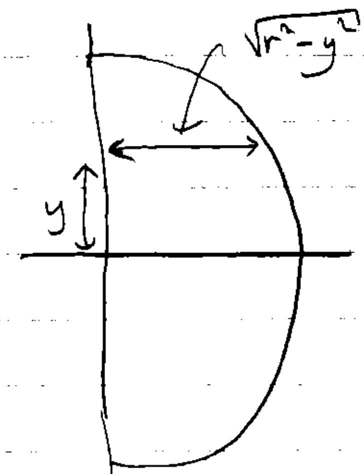
LET'S FIND THE
VOLUME WITH CROSS-SECTIONAL SLICING

- EQUATION FOR PTS (x,y) ON EDGE OF THE CIRCLE $\Rightarrow$ $x^2 + y^2 = r^2$
- CUT PERPENDICULAR TO AXIS OF ROTATION
USE dy!

$$\int_{-r}^{r} \pi \left(\text{PATTERN FOR THE RADIUS OF EACH DISC}\right)^2 dy$$



$$x^2 + y^2 = r^2 \Rightarrow x^2 = r^2 - y^2$$
$$\Rightarrow x = \pm\sqrt{r^2 - y^2}$$



$$\text{VOLUME} = \int_{-r}^{r} \pi \left(\sqrt{r^2 - y^2}\right)^2 dy$$

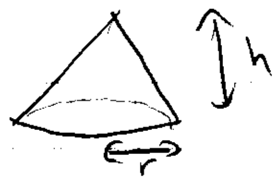
$$= \pi \int_{-r}^{r} r^2 - y^2 \, dy$$

$$= \pi \left[r^2 y - \tfrac{1}{3} y^3 \right] \Big|_{-r}^{r}$$

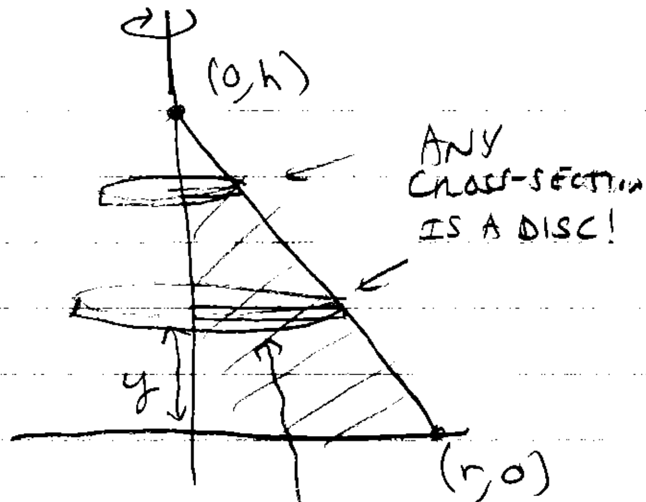
$$= \pi\left[\left(r^3 - \tfrac{1}{3}r^3\right) - \left(-r^3 + \tfrac{1}{3}r^3\right)\right]$$

$$\pi\left[\tfrac{2}{3}r^3 + \tfrac{2}{3}r^3\right] = \boxed{\tfrac{4}{3}\pi r^3} \longleftarrow \text{VOL. OF SPHERE!}$$

2 $\boxed{\text{CONE}}$



CONSIDER THIS REGION
ROTATE ABOUT THE y-AXIS



· EQUATION FOR THE LINE

$$\text{SLOPE} = \frac{h-0}{0-r} = -\frac{h}{r}$$

$$\boxed{y = -\frac{h}{r}x + h}$$

$$\text{So} \quad y - h = -\frac{h}{r}x$$

$$\Rightarrow \quad x = -\frac{r}{h}y - \frac{r}{h}(-h)$$

$$\boxed{x = -\frac{r}{h}y + r}$$

$$\text{VOLUME} = \int_0^h \pi\left(-\frac{r}{h}y + r\right)^2 dy$$

EXPANDING

$$= \pi \int_0^h \frac{r^2}{h^2}y^2 - \frac{2r^2}{h}y + r^2 \, dy$$

$$= \pi\left[\frac{r^2}{h^2}\frac{1}{3}y^3 - \frac{r^2}{h}y^2 + r^2 y \Big|_0^h\right]$$

$$= \pi\left[\frac{1}{3}\frac{r^2}{\cancel{h^2}}h^{\cancel{3}} - \frac{r^2}{\cancel{h}}h^{\cancel{2}} + r^2 h\right]$$

$$= \boxed{\frac{1}{3}\pi r^2 h}$$

VOLUME OF CONE.