1. (10 pts) Evaluate the integrals.

5 (a) $\int 3x^4 + \frac{5}{2x} - \sqrt{\frac{9}{x^3}} + \cos(4x)\,dx$

$$= \int 3x^4 + \frac{5}{2}\,\frac{1}{x} - 3x^{-3/2}\,dx + \int \cos(4x)\,dx$$

$u = 4x$

$du = 4dx$

$\frac{1}{4}du = dx$

$$= \frac{3}{5}x^5 + \frac{5}{2}\ln|x| - 3(-2)x^{-1/2} + \frac{1}{4}\int \cos(u)\,du$$

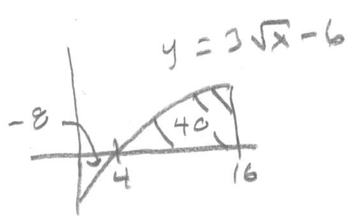
$$= \boxed{\frac{3}{5}x^5 + \frac{5}{2}\ln|x| + \frac{6}{\sqrt{x}} + \frac{1}{4}\sin(4x) + C}$$

5 (b) $\int_0^{16} |3\sqrt{x} - 6|\,dx$

$3\sqrt{x} - 6 \stackrel{?}{=} 0$

$\sqrt{x} = 2$

$x = 4$



$$\int_0^4 3\sqrt{x} - 6\,dx = 3\,\tfrac{2}{3}x^{3/2} - 6x\Big|_0^4 = (2(4)^{3/2} - 6(4)) - 0$$

$$= 16 - 24 = -8$$

$$\int_4^{16} 3\sqrt{x} - 6\,dx = 2x^{3/2} - 6x\Big|_4^{16} = (2(16)^{3/2} - 6(16)) - (-8)$$

$$= 128 - 96 + 8 = 40$$

$$\int_0^{16} |3\sqrt{x} - 6|\,dx = 8 + 40 = \boxed{48}$$

14

2. (15 pts)

4 (a) Evaluate $\displaystyle\int \frac{\sin(t)}{\cos^2(\cos(t))}\,dt$

$u = \cos(t)$

$du = -\sin(t)\,dt$

$\frac{1}{-\sin(t)}\,du = dt$

$$= \int \frac{\cancel{\sin(t)}}{\cos^2(u)} \frac{1}{-\cancel{\sin(t)}}\,du$$

$$= -\int \frac{1}{\cos^2(u)}\,du = -\int \sec^2(u)\,du$$

$$= -\tan(u) + C$$

$$= \boxed{-\tan(\cos(t)) + C}$$

6 (b) Evaluate $\displaystyle\int_1^4 \frac{e^{(1-\sqrt{x})}}{6\sqrt{x}}\,dx$

$u = 1 - \sqrt{x}$

$du = -\frac{1}{2\sqrt{x}}\,dx$

$-2\sqrt{x}\,du = dx$

$$\int_0^{-1} \frac{e^u}{6\cancel{\sqrt{x}}} \, -2\cancel{\sqrt{x}}\,du$$

$$= -\frac{1}{3}\int_0^{-1} e^u\,du = \frac{1}{3}\int_{-1}^{0} e^u\,du = \frac{1}{3} e^u \Big|_{-1}^{0}$$

$$= \boxed{\frac{1}{3}(1-e^{-1}) = \frac{1}{3}\left(1-\frac{1}{e}\right)} \approx 0.21071$$

5 (c) A particular function $y = f(x)$ satisfies the following: $\displaystyle\int_0^6 f(w)\,dw = 12$ and $\displaystyle\int_0^5 f(u)\,du = 4$.

Find the value of $\displaystyle\int_5^6 2f(x) + 10\,dx + \int_0^2 f(3t)\,dt$

$u = 3t$

$du = 3\,dt$

$\frac{1}{3}du = dt$

$$2\underbrace{\int_5^6 f(x)\,dx} + \int_5^6 10\,dx + \frac{1}{3}\int_0^6 f(u)\,du$$

$$= 2(12-4) + 10 + \frac{1}{3}(12)$$

$$= 16 + 10 + 4 = \boxed{30}$$

3. (13 pts)

4

(a) Use the right-endpoint method with $n = 4$ subdivision to approximate $\int_1^3 \ln(2t+1)\,dt$. (Leave your answer expanded out with all the correct numbers in the correct places).

$\Delta t = \frac{3-1}{4} = 1/2, \quad t_0 = 1, \; t_1 = \frac{3}{2}, \; t_2 = 2, \; t_3 = \frac{5}{2}, \; t_4 = 3$

$$\boxed{\ln\left(2\left(\tfrac{3}{2}\right)+1\right)\tfrac{1}{2} + \ln(2(2)+1)\tfrac{1}{2} + \ln\left(2\cdot\left(\tfrac{5}{2}\right)+1\right)\tfrac{1}{2} + \ln(2(3)+1)\tfrac{1}{2} = \tfrac{1}{2}\left(\ln(4)+\ln(5)+\ln(6)+\ln(7)\right)} \approx 3.3667$$

3

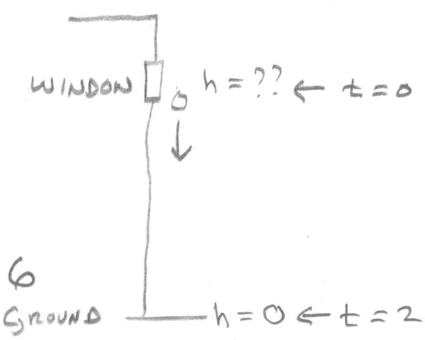
(b) Using the right-endpoint method with n subdivisions, write out the general pattern in terms of i and n for the Riemann sum for $\int_1^3 \ln(2t+1)\,dt$. (*i.e.* fill in the pattern in the space provided after the sigma sign below).

$\Delta t = \frac{2}{n}, \quad t_i = 1 + \frac{2i}{n}$

Answer: $\displaystyle\int_1^3 \ln(2t+1)\,dt = \lim_{n\to\infty} \sum_{i=1}^{n} \boxed{\ln\left(2\left(1+\tfrac{2i}{n}\right)+1\right)\tfrac{2}{n}}$

6

(c) A water balloon is thrown downward from a dorm window. After 2 seconds, the balloon *coincidentally* hits the ground right next your math instructor. Your math instructor estimates the balloon hit the ground at a (downward) speed of 80 ft/sec. At what height is the window from which the balloon was thrown? (Assume acceleration is a constant 32 ft/sec^2 downward).



$a(t) = -32$

$v(t) = -32t + C$

$h(t) = -16t^2 + Ct + D$

$v(2) = -80 \Rightarrow -32(2) + C \stackrel{?}{=} -80 \Rightarrow C = -16$

$h(2) = 0 \Rightarrow -16(2)^2 - 16(2) + D \stackrel{?}{=} 0$

$-64 - 32 + D = 0$

$h(0) = D = \boxed{96 \text{ feet}}$

4. (10 pts) For all parts, consider the region R bounded by $y = \sqrt{x}$, $y = -\sqrt{x}$ and $x = 3y + 10$.

5 (a) Set up an integral for the area of the region R. (Just set up, DO NOT EVALUATE)

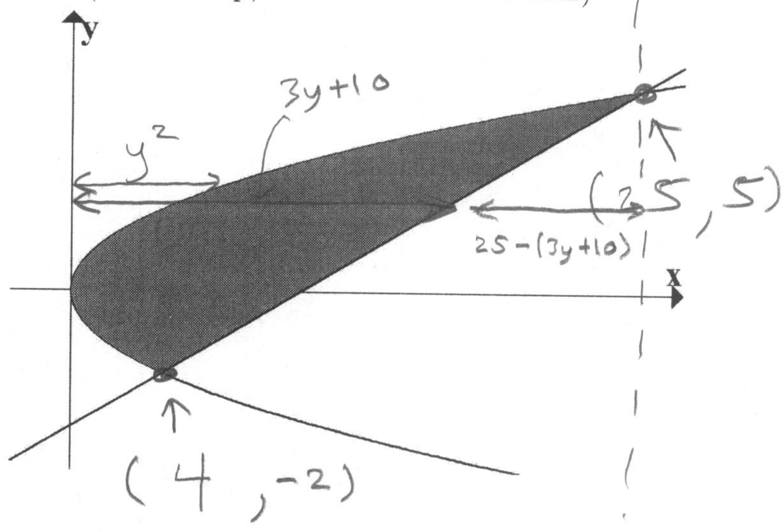
IN TERMS OF y:

$$\left.\begin{array}{l} y = \sqrt{x} \\ y = -\sqrt{x} \end{array}\right\} \quad x = y^2$$

$$y = \frac{x-10}{3} \iff x = 3y + 10$$

INTERSECTIONS:

$$y^2 = 3y + 10 \Rightarrow y^2 - 3y - 10 \stackrel{?}{=} 0$$
$$(y-5)(y+2) = 0$$
$$y = -2,\ y = 5$$



$$\text{AREA} = \boxed{\int_{-2}^{5} 3y + 10 - y^2\, dy}$$

OR

$$\boxed{\int_0^4 \sqrt{x} - -\sqrt{x}\, dx + \int_4^{25} \sqrt{x} - \frac{(x-10)}{3}\, dx}$$

3 (b) Set up an integral that represents the volume of the solid obtained by rotating the region R about the y-axis. (DO NOT EVALUATE)

WASHERS

$$\boxed{\int_{-2}^{5} \pi(3y+10)^2 - \pi(y^2)^2\, dy}$$

3 (c) Set up an integral that represents the volume of the solid obtained by rotating the region R about the **vertical** line $x = 25$. (DO NOT EVALUATE)

WASHERS

$$\boxed{\int_{-2}^{5} \pi(25 - y^2)^2 - \pi(25 - (3y+10))^2\, dy}$$

5. (12 pts) The two parts below are not related.

4 (a) The distance traveled by a bike on a certain track is $F(x) = \int_0^{2x} \sqrt{\frac{1}{3-\cos(t)}}\, dt$ where x is in seconds and $F(x)$ is in feet. Find the speed of the bike at $t = \pi$ seconds (include units).

$$F'(x) = \sqrt{\frac{1}{3-\cos(2x)}} \cdot 2$$

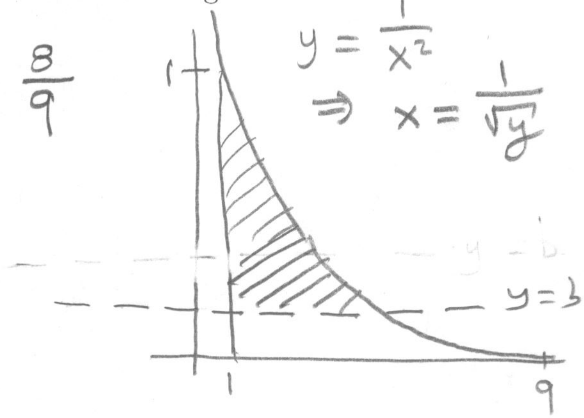
≈ 1.4142

$$F'(\pi) = \sqrt{\frac{1}{3-1}}\; 2 = \boxed{\frac{2}{\sqrt{2}} = \sqrt{2}\ \frac{ft}{sec}}$$

7 (b) Consider the region under the curve $y = \frac{1}{x^2}$, above the x-axis, and between $x = 1$ and $x = 9$. Find the number b such that the **horizontal** line $y = b$ bisects this region.

$$\text{TOTAL AREA} = \int_1^9 \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_1^9 = -\frac{1}{9} - -1 = \frac{8}{9}$$

$$y = \frac{1}{x^2} \Rightarrow x = \frac{1}{\sqrt{y}}$$



$$\text{WANT: } \int_b^1 y^{-1/2} - 1\,dy \overset{?}{=} \frac{1}{2}\,\frac{8}{9}$$

$$2y^{1/2} - y\Big|_b^1 \overset{?}{=} \frac{4}{9}$$

$$(2-1) - (2\sqrt{b} - b) \overset{?}{=} \frac{4}{9}$$

$$\Rightarrow \frac{5}{9} + b = 2\sqrt{b} \quad \Big)\cdot 9$$

$$\Rightarrow 5 + 9b = 18\sqrt{b} \quad \Big) \text{ square}$$

$$25 + 90b + 81b^2 = 324b$$

$$81b^2 - 234b + 25 \overset{?}{=} 0$$

$$b = \frac{234 \pm \sqrt{(234)^2 - 4(81)(25)}}{2(81)} = \frac{234 \pm \sqrt{46656}}{162} = \frac{234 \pm 216}{162}$$

$= \text{OR}$ $\cancel{\frac{450}{162}}$ TOO BIG, or $\frac{18}{162} = \frac{1}{9}$

$$\boxed{b = \frac{1}{9} \approx 0.\overline{1}} = \frac{18}{162}$$