1. (12 points)

(a) Evaluate $\int_1^e \frac{\sqrt{\ln(x)}}{x} dx$.

$u = \ln(x)$

$du = \frac{1}{x} dx$

$dx = x \, du$

$x = 1 \Rightarrow u = 0$

$x = e \Rightarrow u = 1$

$= \int_0^1 \frac{\sqrt{u}}{\cancel{x}} \cancel{x} \, du$

$= \int_0^1 u^{1/2} du$

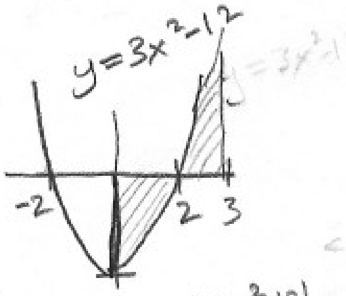
$= \frac{2}{3} u^{3/2} \Big|_0^1 = \boxed{\frac{2}{3}}$

(b) If $g(x) = \int_0^{\ln(x)} \frac{e^t}{1+t} dt$ for $x \geq 1$, find $g'(e)$.

$g'(x) = \frac{e^{\ln(x)}}{1+\ln(x)} \cdot \frac{1}{x} = \frac{\cancel{x}}{(1+\ln(x))\cancel{x}} = \frac{1}{1+\ln(x)}$

$g'(e) = \frac{1}{1+\ln(e)} = \boxed{\frac{1}{2}}$

(c) Evaluate: $\int_0^3 |3x^2 - 12| \, dx$

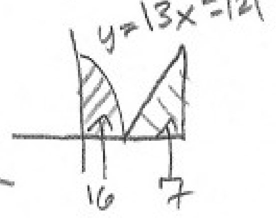


$3x^2 - 12 = 0$

$\Rightarrow 3x^2 = 12$

$x^2 = 4$

$x = \pm 2$



$\int_0^2 3x^2 - 12 \, dx = x^3 - 12x \Big|_0^2$

$= 8 - 24$

$= -16$

$\int_2^3 3x^2 - 12 \, dx = x^3 - 12x \Big|_2^3$

$= (27-36) - (8-24)$

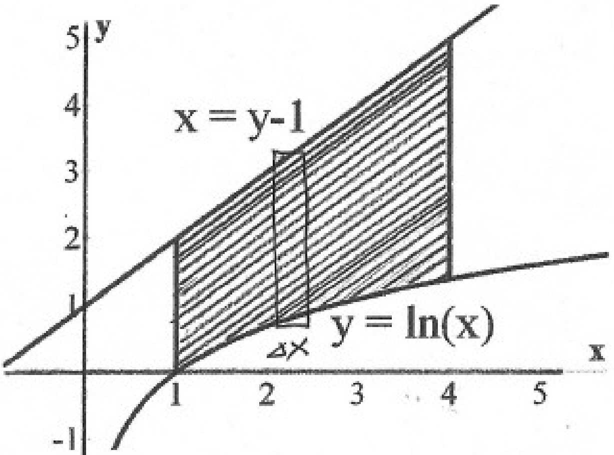
$= -9 - (-16) = 7$

$16 + 7$

$= \boxed{23}$

2. (8 points)

Consider the region R bounded by the curves $y = \ln(x)$, $x = y - 1$, $x = 1$, and $x = 4$. A picture of this region is given at right.



(a) Set up an integral that represents the **area** of the region R. (Do not evaluate the integral.)

In terms of x is much easier

$y = x+1$

$y = \ln(x)$

$$\boxed{\text{AREA} = \int_1^4 x+1-\ln(x)\,dx}$$

Aside

In terms of y, it becomes

$$\int_0^{\ln(4)} e^y - 1\,dy + \int_{\ln(4)}^{2} 4-1\,dy + \int_2^5 4-(y-1)\,dy$$

(b) Approximate the **area** of this region using $n = 3$ approximating rectangles and right endpoints.

$\Delta x = 1$

$$R_3 = (2+1-\ln(2))\cdot 1 + (3+1-\ln(3))\cdot 1 + (4+1-\ln(4))\cdot 1$$

$$= 3-\ln(2) + 4-\ln(3) + 5-\ln(4)$$

$$= 12 - \ln(2) - \ln(3) - \ln(4) = 12-\ln(24)$$

$$\approx \boxed{8.8219}$$

3. (12 points) Evaluate the following integrals:

(a) $\int \frac{(1+x)\sqrt{x}}{x}dx$ $= \int \frac{x^{1/2}+x^{3/2}}{x} dx$

$= \int x^{-1/2} + x^{1/2}\, dx$

$= \boxed{2x^{1/2} + \frac{2}{3}x^{3/2} + C}$

(b) $\int \frac{\sin(\sqrt[3]{x})}{x^{2/3}} + \cos(x)\, dx$ $= \underbrace{\int \frac{\sin(x^{1/3})}{x^{2/3}} dx}$ $+ \underbrace{\int \cos(x) dx}_{\sin(x) + C_0}$

$\int \frac{\sin(u)}{\cancel{x^{2/3}}} 3\cancel{x^{2/3}} du$

$u = x^{1/3}$

$du = \frac{1}{3}x^{-2/3} dx$ $\quad du = \frac{1}{3x^{2/3}} dx$

$dx = 3x^{2/3} du$

$= 3\int \sin(u) du$

$= -3\cos(u) + C_1$

$\boxed{\text{ANSWER} = -3\cos(x^{1/3}) + \sin(x) + C}$

(c) $\int x^3(1+x^2)^{10}dx$

$u = 1 + x^2$ $\quad x^2 = u - 1$

$du = 2x\, dx$

$dx = \frac{1}{2x} du$

$\int x^{\cancel{3}\,2} u^{10} \frac{1}{2\cancel{x}} du$

$\frac{1}{2}\int x^2 u^{10} du$

$\frac{1}{2}\int (u-1)u^{10} du = \frac{1}{2}\int u^{11} - u^{10} du$

$= \frac{1}{2}\left[\frac{1}{12}u^{12} - \frac{1}{11}u^{11}\right] + C$

$= \boxed{\frac{1}{24}(1+x^2)^{12} - \frac{1}{22}(1+x^2)^{11} + C}$

4. (8 points) Suppose you look out the window of a skyscraper and see someone throw an apple downward. Your window is at a height of 370 feet. The apple passes your window after 3 seconds (from the time it was thrown). The velocity at 3 seconds is -100 feet per second. Assuming that the apple has a constant acceleration of $a(t) = -32$ ft/sec^2, answer the following questions.

(a) Give the formula for the position of the apple at time, t, seconds after being thrown. (You should determine the values of all constants.)

$a(t) = -32$

$v(3) = -100$

$s(3) = 370$

$v(t) = -32t + C_0$

$-100 = -32(3) + C_0$

$\Rightarrow C_0 = -4$

$v(t) = -32t - 4$

$s(t) = -16t^2 - 4t + C_1$

$370 = -16(3)^2 - 4(3) + C_1$

$C_1 = 526$

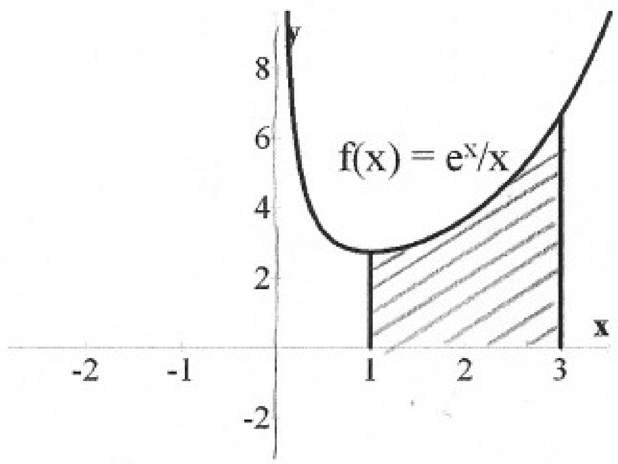
$$\boxed{s(t) = -16t^2 - 4t + 526}$$

(b) Find the velocity at which the apple was thrown and also the height from which it was thrown.

$$\boxed{\begin{array}{l} v(0) = -4 \text{ ft/sec} \\ s(0) = 526 \text{ feet} \end{array}}$$

5. (10 points)

Consider the region R bounded by $f(x) = \dfrac{e^x}{x}$, $x = 1$, $x = 3$ and the x-axis. A picture of this region is given at right.



(a) Set up an integral of the form $\int_a^b f(x)dx$ that represents the **volume** of the solid obtained by rotating the region, R, about the line $y = -2$.
(Do not evaluate the integral.)

Slicing $\int_a^b \text{Area}\, dx$

$$\int_1^3 \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2\, dx$$

$$\boxed{\int_1^3 \pi\left(\frac{e^x}{x} + 2\right)^2 - \pi(2)^2\, dx}$$

(b) Find the exact **volume** of the solid obtained by rotating the region, R, about the y-axis.
(Set up and evaluate the integral.)

Shells $\int_a^b 2\pi(\text{radius})(\text{height})\, dx$

$$\int_1^3 2\pi \cancel{x} \frac{e^x}{\cancel{x}}\, dx$$

$$2\pi \int_1^3 e^x\, dx = 2\pi\left[e^x \Big|_1^3\right]$$
$$= \boxed{2\pi(e^3 - e)}$$