MATH 125 EXAM 1 SOL'N FALL 2009

① (a) $\int_1^4 (5\sqrt{x}+2)x\,dx = \int_1^4 5x^{3/2}+2x\,dx$

$= 5\,\frac{2}{5}x^{5/2}+x^2\Big|_1^4$

$= \left[2(4)^{5/2}+(4)^2\right]-\left[2(1)^{5/2}+(1)^2\right]$

$= 2(2)^5+16-2-1$

$= 64+16-2-1 = 80-3 = \boxed{77}$

(b) $\int \frac{x^5}{1+x^3}\,dx$

$= \int \frac{x^5}{u}\frac{1}{3x^2}\,du$

$= \frac{1}{3}\int \frac{x^3}{u}\,du$

$= \frac{1}{3}\int \frac{u-1}{u}\,du$

$= \frac{1}{3}\int 1-\frac{1}{u}\,du = \frac{1}{3}(u-\ln|u|)+C$

$= \boxed{\frac{1}{3}(1+x^3)-\frac{1}{3}\ln|1+x^3|+C}$

$u=1+x^3 \qquad x^3=u-1$

$du=3x^2dx$

$dx=\frac{1}{3x^2}du$

(c) $\underbrace{\int \sin(x)\sec(\cos(x))\tan(\cos(x))dx}+\underbrace{3\int e^x\,dx}_{3e^x}+C_2$

$\int \sin(x)\sec(u)\tan(u)\frac{1}{-\sin(x)}\,du$

$-\int \sec(u)\tan(u)\,du$

$= -\sec(u)+C_1$

$= -\sec(\cos(x))$

$u=\cos(x)$

$du=-\sin(x)dx$

$dx=\frac{1}{-\sin(x)}du$

$\boxed{-\sec(\cos(x))+3e^x+C}$

② $A(x) = \int_1^{x^2} \frac{t-4}{t+7}\,dt \Rightarrow A'(x) = \left(\frac{x^2-4}{x^2+7}\right) 2x$

$A'(x) = 0 \Leftrightarrow \frac{(x^2-4)2x}{x^2+7} = 0 \Leftrightarrow (x^2-4)2x = 0 \Leftrightarrow 2(x-2)(x+2)x = 0$

$\boxed{x=0,\ x=2,\ x-2}$

③ (a) $v(t) = \int a(t)\,dt = \int 2t-4\,dt = t^2-4t+C$

$v(2) = -9 \Rightarrow -9 = (2)^2 - 4(2) + C$

$\Rightarrow -9 = 4 - 8 + C$

$\Rightarrow -9 = -4 + C \Rightarrow C = -5$

$\boxed{v(t) = t^2-4t-5}$ $\qquad (t-5)(t+1)$

(b) TOTAL DISTANCE $= \int_0^{10} |t^2-4t-5|\,dt$

$t^2-4t-5=0 \Leftrightarrow (t-5)(t+1)=0 \Leftrightarrow t=-1,\ t=5$

$\int_0^5 t^2-4t-5\,dt = \frac{1}{3}t^3 - 2t^2 - 5t \Big|_0^5$

$= \left(\frac{1}{3}(5)^3 - 2(5)^2 - 5(5)\right) - (0) = -\frac{100}{3} = -33.\overline{3}$

$\int_5^{10} t^2-4t-5\,dt = \frac{1}{3}t^3 - 2t^2 - 5t \Big|_5^{10}$

$= \left(\frac{1}{3}(10)^3 - 2(10)^2 - 5(10)\right) - \left(\frac{1}{3}(5)^3 - 2(5)^2 - 5(5)\right)$

$= 83.\overline{3} - -33.\overline{3} = \frac{250}{3} - -\frac{100}{3}$

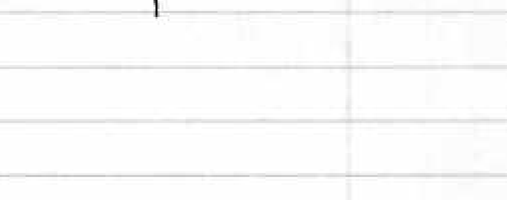
$= 116.\overline{6} = \frac{350}{3}$

$\int_0^{10} |t^2-4t-5|\,dt = 33.\overline{3} + 116.\overline{6} = \frac{100}{3} + \frac{350}{3}$

$= 150 = \frac{450}{3}$

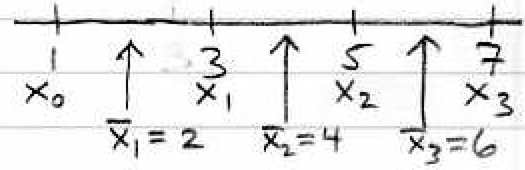
$\boxed{150 \text{ ft}}$

ASIDE



④ $\int_1^7 x\ln(x)\,dx \approx M_3 = f(\bar{x}_1)\Delta x + f(\bar{x}_2)\Delta x + f(\bar{x}_3)\Delta x$

$\Delta x = \frac{7-1}{3} = 2$



$M_3 = (2\ln(2))\,2 + (4\ln(4))\,2 + (6\ln(6))\,2$

$= 4\ln(2) + 8\ln(4) + 12\ln(6) \approx 35.36405724$

$\boxed{35.3641}$

⑤ (a) $\int_0^{5\pi/2} 10\sin\left(\frac{x}{5}\right)dx$

$u = \frac{x}{5}$ $\quad x=0 \Rightarrow u=0$

$du = \frac{1}{5}dx$ $\quad x = \frac{5\pi}{2} \Rightarrow u = \frac{\pi}{2}$

$dx = 5\,du$

$= 10\int_0^{\pi/2} \sin(u)\,5\,du$

$= -50\cos(u)\Big|_0^{\pi/2} = -50\cos\left(\frac{\pi}{2}\right) - -50\cos(0) = 0 + 50$

$\boxed{\text{50 square yards}}$

ⓑ $\int_0^a 10\sin\left(\frac{x}{5}\right)dx = \frac{1}{2}(50)$ Find a.

$u = x/5$ $\quad x=0 \Rightarrow u=0$

$du = \frac{1}{5}dx$ $\quad x=a \Rightarrow u = a/5$

$dx = 5\,du$

$10\int_0^{a/5} \sin(u)\,5\,du = 25$

$-50\cos(u)\Big|_0^{a/5} = 25$

$-50\cos(a/5) - -50\cos(0) = 25$

$-50\cos(a/5) + 50 = 25$

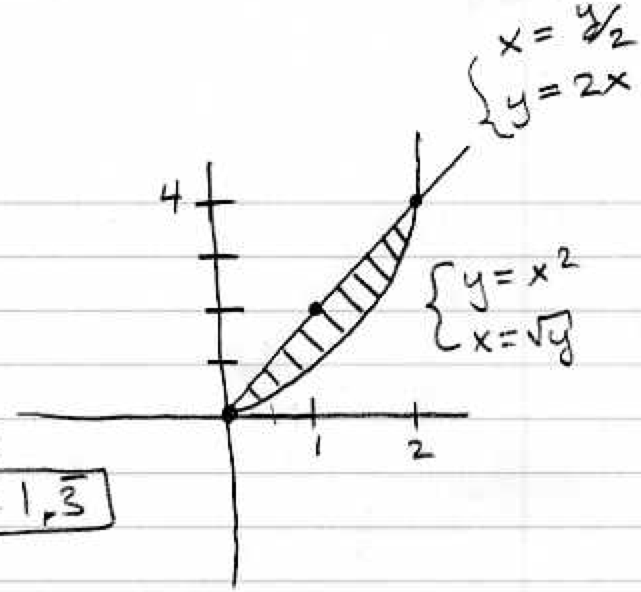
$-50\cos(a/5) = -25$

$\cos(a/5) = 1/2$

$a/5 = \pi/3$

$\boxed{a = 5\pi/3}$ vertical line at $x = 5\pi/3$

(6) (a)



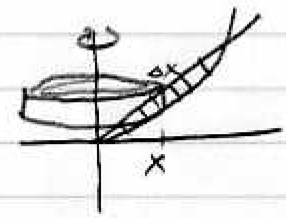
$\boxed{\text{with respect to } x}$

$$\text{AREA} = \int_0^2 2x - x^2\,dx$$
$$= x^2 - \tfrac{1}{3}x^3 \Big|_0^2$$
$$= \left[(2)^2 - \tfrac{1}{3}(2)^3\right] - \left[(0)^2 - \tfrac{1}{3}(0)^3\right]$$
$$= 4 - \tfrac{8}{3} = \tfrac{12}{3} - \tfrac{8}{3} = \boxed{\tfrac{4}{3} = 1.\overline{3}}$$

$\boxed{\text{with respect to } y}$

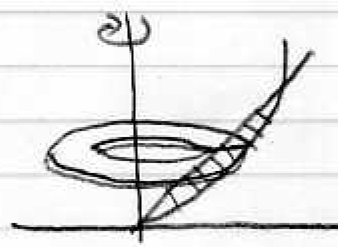
$$\text{AREA} = \int_0^4 \sqrt{y} - y/2\,dy$$
$$= \tfrac{2}{3}y^{3/2} - \tfrac{1}{4}y^2 \Big|_0^4 = \left[\tfrac{2}{3}(4)^{3/2} - \tfrac{1}{4}(4)^2\right] - \left[\tfrac{2}{3}(0)^{3/2} - \tfrac{1}{4}(0)^2\right]$$
$$= \tfrac{2}{3}\,8 - 4 = \tfrac{16}{3} - \tfrac{12}{3} = \boxed{\tfrac{4}{3} = 1.\overline{3}}$$

(b) $\boxed{\text{with respect to } x : \text{SHELLS}}$



$$V = \int_0^2 2\pi\,(\text{RADIUS})(\text{HEIGHT})\,dx$$
$$\int_0^2 2\pi x\,(2x - x^2)\,dx$$
$$= 2\pi \int_0^2 2x^2 - x^3\,dx = 2\pi\left[\tfrac{2}{3}x^3 - \tfrac{1}{4}x^4 \Big|_0^2\right]$$
$$= 2\pi\left[\left(\tfrac{2}{3}(2)^3 - \tfrac{1}{4}(2)^4\right) - (0)\right] = 2\pi\left[\tfrac{16}{3} - 4\right] = 2\pi\,\tfrac{4}{3} = \boxed{\tfrac{8\pi}{3}}$$

$\boxed{\text{with respect to } y : \text{WASHER}}$



$$V = \int_0^4 \pi(\text{OUTER})^2 - \pi(\text{INNER})^2\,dy$$
$$= \int_0^4 \pi\left(\sqrt{y}\right)^2 - \pi\left(\tfrac{y}{2}\right)^2 dy$$
$$= \pi \int_0^4 y - \tfrac{1}{4}y^2\,dy = \pi\left[\tfrac{1}{2}y^2 - \tfrac{1}{12}y^3 \Big|_0^4\right] \qquad \tfrac{24}{3}$$
$$= \pi\left[\left(\tfrac{1}{2}(4)^2 - \tfrac{1}{12}(4)^3\right) - (0)\right] = \pi\left[\tfrac{1}{2}16 - \tfrac{16}{12}64\right] = \pi\left[8 - \tfrac{16}{3}\right]$$
$$= \boxed{8\pi/3}$$

(7) (a)

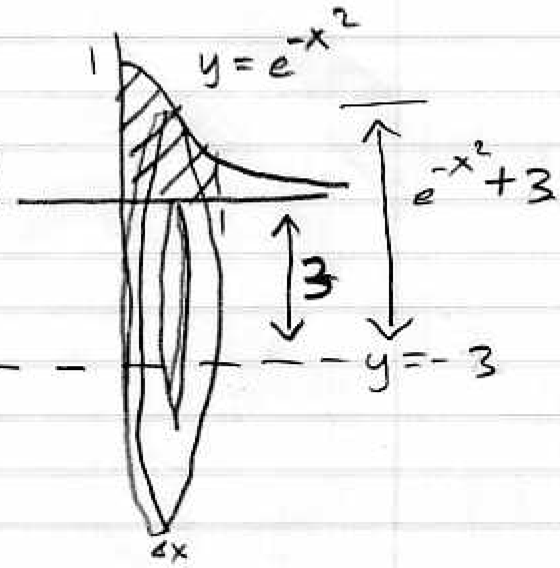
VOLUME $= \int_0^1 \pi(\text{OUTER})^2 - \pi(\text{INNER})^2 \, dx$

$= \boxed{\int_0^1 \pi\left(e^{-x^2}+3\right)^2 - \pi(3)^2 \, dx}$

Also can be simplified to get

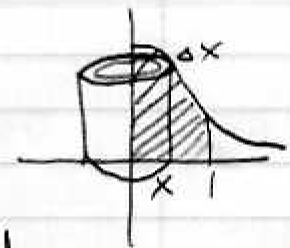
$\pi \int_0^1 e^{-2x^2} + 6e^{-x^2} + \cancel{9} - \cancel{9} \, dx$

$\pi \int_0^1 e^{-2x^2} + 6e^{-x^2} \, dx$



(b) VOLUME $= \int_0^1 2\pi(\text{RADIUS})(\text{HEIGHT}) \, dx$

$= \int_0^1 2\pi x \, e^{-x^2} \, dx$



$2\pi \int_0^{-1} \cancel{x} \, e^u \frac{1}{-2\cancel{x}} \, du$

$u = -x^2$

$du = -2x \, dx$

$dx = \frac{1}{-2x} \, du$

$x=0 \Rightarrow u=0$

$x=1 \Rightarrow u=-1$

$-\pi \int_0^{-1} e^u \, du$

$-\pi\left[e^u \Big|_0^{-1}\right] = -\pi\left[e^{-1} - e^0\right]$

$= \boxed{-\pi\left[e^{-1} - 1\right] = \pi(1-e^{-1}) = \pi\left(1 - \frac{1}{e}\right)}$