BASIC REVIEW OF CALCULUS I

This review sheet discuss some of the key points of Calculus I that are essential for understanding Calculus II. This review is not meant to be all inclusive, but hopefully it helps you remember basics. Please notify me if you find any typos on this review sheet.

1. By now you should be a derivative expert. You should be comfortable with derivative notation ($f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}$). Here is some of the main derivative rules and concepts that I expect you to know:

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$	$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\frac{d}{dx}(\cos(x)) = -\sin(x)$	$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$	$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$	$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$	$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{x^2+1}$
$(FS)' = FS' + F'S$	$\left(\frac{N}{D}\right)' = \frac{DN' - ND'}{D^2}$	$[f(g(x))]' = f'(g(x))g'(x)$

2. The derivative of a function at a point represents the slope of the tangent line at that point. That is,

$$f'(a) = \text{“the slope of the tangent line to the graph of } f(x) \text{ at } x = a.\text{”}$$

We can use this to help study the behavior and the graph of $f(x)$.

   (a) $f'(x) > 0$ exactly when $f(x)$ is increasing.
   (b) $f'(x) < 0$ exactly when $f(x)$ is decreasing.
   (c) $f'(x) = 0$ exactly when $f(x)$ has a horizontal tangent.

3. The second derivative of a function at a point represents the concavity of the graph at that point. A function is *concave up* at $x = a$ if the graph is above the tangent line near the point $x = a$. A function is *concave down* at $x = a$ if the graph is below the tangent line near the point $x = a$. We use the second derivative in the following way.

   (a) $f''(x) > 0$ exactly when $f(x)$ is concave up.
   (b) $f''(x) < 0$ exactly when $f(x)$ is concave down.
   (c) A *point of inflection* is a location where concavity changes. Note that $f''(x) = 0$ does not necessarily mean that we have a point of inflection.

4. In many applications, we want to find local maximum and local minimum values. In these applications, it makes sense to look at the locations on a graph where the slope is 0. In other words, we often use the following method to find all locations where the slope of the graph is 0:

   (a) Find the formula for the derivative: $f'(x)$.
   (b) Set this formula equal to 0 and solve for x.

- If $f'(x) = 0$ and $f''(x) > 0$ at a point, then the point corresponds to a local minimum.
- If $f'(x) = 0$ and $f''(x) < 0$ at a point, then the point corresponds to a local maximum.

5. You should understand limits; How to compute them and how they relate to calculus. When computing limits it may be useful to try the following techniques:

   (a) If the function is continuous at the point, just plug in the point.

   (b) If it is not continuous at the point try the following:
   - Simplify by factoring or rationalizing.
   - Write in the form "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$" and apply L'Hospital's Rule. That is, take the derivative of the top and bottom separately and then try to take the limit.

   A major application of limits in Calculus I comes from the definition of the derivative. In particular, we defined the derivative of a function $f(x)$ to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

6. A common problem for calculus students is remembering the properties of trigonometric and logarithmic functions. I review some of these key ideas here:

   - Given a right triangle and a non-right angle, θ, in the triangle, you should understand what is meant by

$\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$	$\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	$\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}} = \frac{\sin(\theta)}{\cos(\theta)}$
$\csc(\theta) = \frac{1}{\sin(\theta)}$	$\sec(\theta) = \frac{1}{\cos(\theta)}$	$\cot(\theta) = \frac{1}{\tan(\theta)}$

   You also should know all the following basic trigonometric values. Note that you can use these values along with the rules above and the symmetry of locations on a circle to find several other values of the trigonometric functions.

x	$\sin(x)$	$\cos(x)$
0	0	1
$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\pi/2$	1	0

   - Recall the basic trig identities. The first identity should be very familiar, you can get the others in the first row by dividing the first identity by $\cos^2(x)$ and $\sin^2(x)$, respectively. The second row contains the so-called "double angle identities". We will make use of these during this course.

$\sin^2(x) + \cos^2(x) = 1$	$\tan^2(x) + 1 = \sec^2(x)$	$1 + \cot^2(x) = \csc^2(x)$
$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$	$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$	$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$

   - Finally remember the basic $\ln(x)$ and power rules:

$\ln(1) = 0$	$\ln(e) = 1$	$\ln(a^b) = b\ln(a)$	$\ln(ab) = \ln(a) + \ln(b)$
$x^a x^b = x^{a+b}$	$\frac{x^a}{x^b} = x^{a-b}$	$\frac{1}{x^a} = x^{-a}$	$(x^a)^b = x^{ab}$

   - There is no way to simplify $\sqrt{a^2+b^2}$ in general. In particular, $\sqrt{a^2+b^2} \neq a+b$. Notice that $\sqrt{3^2+4^2} = \sqrt{25} = 5$, this is not the same at $\sqrt{3^2} + \sqrt{4^2} = 7$. DO NOT MAKE THE MISTAKE OF SIMPLIFYING $\sqrt{a^2+b^2}$.