Closing Wed: HW9A, 9B (9.3, 9.4)
Final: March 10\textsuperscript{th}, 1:30-4:20 in KANE 210
Comprehensive (8-10 pages).
There \textbf{will} be two pages on ch 9.

Ch. 9: Be able to
1. Solve separable diff. eq.
2. Use initial conditions & constants.
3. Set up and do ALL the applied problems from homework.

\textbf{Worried about applied problems?}
Pay attention today and Monday in lectures. Know the homework well.
And go thru my review sheets and look at old finals.

\textit{Newton’s Cooling Law Experiment}
Hot water is in the cup. We will try to predict the temp. at the end of class.
1\textsuperscript{st} measurement:
\begin{align*}
\text{Time} &= \\
\text{Temp} &= 
\end{align*}
2\textsuperscript{nd} measurement:
\begin{align*}
\text{Time} &= \\
\text{Temp} &= 
\end{align*}
9.4 Differential Equations Applications

1. Law of Natural Growth/Decay:
   Assumption: "The rate of growth/decay is proportional to the function value."
   \[
   \frac{dP}{dt} = kP \quad \text{with} \quad P(0) = P_0
   \]

   Example:
   A pop. has 500 bacteria at $t=0$.
   After 3 hrs there are 8000 bacteria.
   Assume the pop. grows at a rate proportional to its size. Find $B(t)$. 
Example:
The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. Find \( m(t) \).

*Example:*
Bob deposits $2000 into a savings account. The money grows at a rate proportional to its size (*i.e.* compound interest). The balance in 4 years is $2100. Find the formula \( A(t) \) for the amount in his account in \( t \) years.
2. Newton’s Law of Cooling:
Assumption: “The rate of temperature change is proportional to the difference between the temperature of the object and its surroundings.”
3. Mixing Problems:
Assume a vat of water has a contaminant entering at some rate and exiting at some rate, then
“The rate of change of the contaminant is equal to the rate at which the contaminant is coming IN minus the rate at which it is going OUT.”
Example:
A 10 Liter vat contains 2kg of salt.
Salt water (brine) containing 3kg of salt per liter enters the vat at 5 L/min.
The vat is well mixed.
The mixture leaves the vat at 5L/min.
Let \( y(t) = \) kilograms of salt in the vat at time \( t \).
   (a) Find \( y(t) \).
   (b) Find the limit of \( y(t) \) as \( n \to \infty \).
**Key Notes:**

- \( V \) = volume of the vat (liters)
- \( t \) = time (min)
- \( y(t) \) = amount in vat (kg)
- \( \frac{dy}{dt} \) = rate (kg/min)

\[
\frac{dy}{dt} = \text{Rate In} - \text{Rate out}
\]

\[
= \left( ? \ \frac{kg}{L} \right) \left( ? \ \frac{L}{min} \right) - \left( \frac{y}{V} \ \frac{kg}{L} \right) \left( ? \ \frac{L}{min} \right)
\]

\[
y(0) = ? \ \text{kg}
\]
**Example:**

A 100 Liter vat contains 5kg of salt.
Two pipes (A & B) pump in salt water.

*Pipe A:* 3L/min with 4kg of salt/L.
*Pipe B:* 5L/min with 2kg of salt/L.
The mixture leaves the vat at 8L/min.

Let $y(t) =$ the amount of salt in the vat at time $t$.

(c) Find $y(t)$.
(d) Find the limit of $y(t)$ as $n \to \infty$. 

4. **Air Resistance:**
A skydiver steps out of a plane that is 4,000 meters high with and initial downward velocity of 0 m/s. The skydiver has a mass of 60 kg. (Treat downward as positive).

Let $y(t) = \text{“height at time } t\text{”}$
Let $v(t) = y'(t) = \text{“velocity at time } t\text{”}$
Let $a(t)=v'(t)=''(t) = \text{“accel. at time } t\text{”}$

**Newton’s 2nd Law** says:

$$(\text{mass})(\text{acceleration}) = \text{Force}$$

$m \frac{d^2 y}{dt^2} = \text{sum of forces on the object}$

The force due to gravity has constant magnitude (and it is acting downward):

$F_g = mg = 60 \cdot 9.8 = 588 \text{ N}$

*One model for air resistance*

The force due to air resistance (*drag force*) is proportional to velocity and in the opposite direction of velocity. So $F_d = -k v$ Newtons

Assume for this problem $k = 12$. 
The Logistics Equation
Consider a population scenario where there is a limit to the amount of growth (spread of a rumor, for example).

Let $P(t) =$ population size at time $t$.

$M =$ maximum population size.

(capacity)

We want a model that
...is like natural growth when $P(t)$ is significantly smaller than $M$;
...levels off (with a slope approaching zero), then the population approaches $M$.

One such model is the so-called logistics equation

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{M}\right) \text{ with } P(0) = P_0$$
Random old final questions:

**Spring 2011 Final:**

Brief summary of what it says:

\( v(t) = \) velocity of an object

\[
F = mg - kv
\]

Recall:

\[
F = ma = m \frac{dv}{dt}
\]

You are given \( m, g, \) and \( k \) and asked for solve for \( v(t) \).
Spring 2014:
A lake has a volume of 1000 cubic meters and contains pure water. A farm near the lake begins using a pesticide. Runoff from the farmland into the lake is 10 cubic meters of water per day, with a concentration of 50 grams of pesticide per cubic meter of water. The lake drains to the ocean at a rate of 10 cubic meters per day.
Winter 2011
Your friend wins the lottery, and
gives you $P_0$ dollars to help you pay
your college expenses. The money is
invested in a savings account that
earns 10% annual interest,
compounded continuously, and you
withdraw the money continuously (a
pretty good approximation to reality
if you make regular frequent
withdrawals) at a rate of $3600 per
year.
Fall 2009
The swine flu epidemic has been modeled by the Gompertz function, which is a solution of
\[
\frac{dy}{dt} = 1.2 \ y \ (K - \ln(y)),
\]
where \(y(t)\) is the number of individuals (in thousands) in a large city that have been infected by time \(t\), and \(K\) is a constant.

Time \(t\) is measured in months, with \(t = 0\) on July 9, 2009.
On July 9, 2009, 75 thousand individuals had been infected. One month later, 190 thousand individuals had been infected.
1. 500 bacteria are in a dish at $t=0$ hr.
8000 bacteria are in the dish at $t=3$ hr.
Assume the population grows at a rate proportional to its size.
Find the function, $B(t)$, for the bacteria population with respect to time.
2. The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. The mass decays at a rate proportional to its size. Find the function, \( m(t) \), for the mass with respect to time.
3. You invest $10,000 into a savings account and never make any deposits or withdrawals. The balance grows at a rate proportional to its size (i.e. *interest* is a percentage of the balance at any time). In 3 years, you notice your balance is $10,400. Find the function, $A(t)$, for the amount of money in the account with respect to time.