

Work Examples

Here is every type of work problem I could find in the old final exam archive. Try these for more practice.

Springs

- Spr 07 Hooke's law states that the force required to maintain a spring stretched x units beyond its natural length is proportional to x . The work required to stretch the spring from 2 feet beyond its natural length to 4 feet beyond its natural length is 18ft-lb. How far beyond its natural length can the spring be stretched with a force not exceeding 24 pounds?
- Spr 08 A spring has a natural length of 10 cm. The spring is now allowed to hang vertically, with the top end attached to a rigid support and the other end attached to a mass of 1 kg. This causes the spring is stretched 3 cm, to a length of 13 cm.
- Find the spring constant k . The acceleration due to gravity is 9.8 m/sec^2 .
 - A small child pulls down on the mass, stretching the string to 15 cm. How much work does the child do?

Lifting a Chains/Cables

- Fall 06 An 8 foot chain weighs 120 pounds. A large robot is holding one end of the chain 3 feet above the ground, so that 5 feet of the chain are on the ground. How much work must the robot do to lift this end of the chain from a height of 3 feet to a height of 13 feet (so he lifts up 10 feet)?
- Spr 10 True/False: A 10 foot rope weighing 4 pounds is hanging from the ceiling. The work done in pulling the whole rope up to the ceiling is less than 40 ft-lb.
- Fall 10 Water is drawn from a well that is 35 meters deep using a leaky bucket that initially scoops up 20 kilograms of water from the bottom of the well. The mass of the bucket itself is 2 kilograms and the mass of the rope that is attached to the bucket is 0.2 kg/m. The rope is being pulled at a constant rate of 0.5 m/s. The bucket has a hole in it and water leaks from the bucket at a rate of 0.1 kg/s.
- Let y be the height (in meters) of the bucket above the bottom of the well. What is the mass of the water in the bucket when the bucket is y meters high?
 - The acceleration due to gravity is 9.8 m/s^2 . Find the work done when the bucket is pulled from the bottom to the top of the well.
- Spr 12 A 50-foot rope weighs 2 pounds per foot. One end of it has been lifted to a window 15 feet above the ground and the rest is lying coiled on the ground. What is the work needed to pull the whole rope through the window?

Pumping

- Fall 05 A tank has the shape of an open-top hemisphere with radius 10 m that is full of water with density 1000 kg/m^3 . Set up an integral which computes the work required to empty the tank by pumping all of the water to the top of the tank. DO NOT EVALUATE THIS INTEGRAL.
- Spr 06 A small circular pool has a radius of 10 ft, the sides are 3 ft high, and the depth of the water is 2 ft. How much work (in ft-lb) is required to pump all of the water out over the side of the pool? (Water weighs 62.5 lb/ft^3 .)

- Win 07 A tank full of water is on the surface of Mars, where the gravitational acceleration is 3.7m/sec^2 . The tank is in the shape of the curve $y = \frac{1}{3}x^2 - 3$ (the part below the x -axis) rotated around the y -axis; the units are meters. Find the work required to pump the water out of an outlet at the top of the tank, which is at the level of the x -axis. Recall that the density of water is 1000kg/m^3 . Give your answer in decimal form.
- Fall 07 A conical tank is 4 meters high and has a radius of 1 meter at the top (see picture). The bottom 3 meters are full of water. How much work is required to pump out all the water over the rim of the tank? (Recall that the mass density of water is 1000kg/m^3 and the acceleration due to gravity is 9.8m/sec^2 .)

Other Work Questions

- Fall 08 A worker has to tighten a bolt using a wrench. She holds the wrench 30 cm away from the bolt. The more she tightens the bolt, the more force she has to use. The force she applies is equal to $3 + \tan^2 \theta$ newtons, where θ is the angle between the original position of the wrench and the current position, in radians. How much work does she do to turn the bolt $1/8$ of a full turn (that is, $\pi/4$ radians)? Give your answer in joules (1 joule = 1 newton-meter). (Hint: The distance traveled along a circle of radius r in moving an angle of $\Delta\theta$ radians is $r\Delta\theta$.)
- Win 09 You want to dig a hole in the ground in the shape of an inverted circular cone with height 2 m and base radius 2 m. The dirt in the hole has density $\rho = 1676\text{kg/m}^3$, and the acceleration due to gravity is $g = 9.8\text{m/sec}^2$.
- Find the work required to remove the *top* 1 meter of dirt from the hole (moving it up to ground level). Give your answer in decimal form.
 - Find the work required to remove the *bottom* 1 meter of dirt (moving it up to ground level). Give your answer in decimal form.
- Fall 09 The electric force (in Newtons) acting on a charged particle A as a result of the presence of a second charged particle B is given by Coulomb's Law

$$F = \frac{kq_A q_B}{r^2},$$

where r is the distance (in meters) between the particles, q_A and q_B are the charges of A and B in Coulombs, and $k = 9 \times 10^9$ is a constant.

Assume that two particles A and B have opposite charges, with $q_A = 1$ Coulomb and $q_B = -1$ Coulomb. (The force F is negative, indicating that the particles are attracting each other.) Assume that particle A is kept fixed, and that the initial distance between the two particles is 1 meter.

- Find the work done to move particle B from its initial position to a position 2 meters away from particle A .
 - Find the work done to move particle B from its initial position to an infinite distance away from particle A .
- Fall 11 A flat math billboard is in the shape of (what else?) a parabola. Its top side is 6 feet wide and the billboard is 18 feet high, measured from the lowest to the highest point. It is mounted on a pole and the lowest point of the billboard is 15 feet above the ground.
- Before it was mounted on the pole, the billboard was originally lying flat on the ground. The billboard weighs 3 pounds per square foot. Set up a definite integral for the work done in lifting this billboard up to where it now stands. Evaluate the integral and find the work done.
- (Hint: Slice the billboard in strips parallel to the straight edge.)

Fall 12 The Great Pyramid of Giza is believed to be built as a tomb for the Egyptian Pharaoh Khufu. It is thought that when it was built, the side of the square base was 230 meters and it was 147 meters tall. Let $s(y)$ be the length of a side of its square horizontal cross-section at height y meters above the ground. A triangular vertical cross-section is pictured in detail on the right.

Set up and evaluate an integral to find the work done to build this pyramid. Assume that the rocks used to build the pyramid all started at ground level and had to be lifted into place as the pyramid was built, and the rocks fit together with no air between them (and no secret chambers!). Take the acceleration due to gravity to be 9.8 m/s^2 and the density of the rocks to be 2360 kg/m^3 .