

$$1 \int \frac{y^3 - 4y + 7}{y^2 + 2y - 3} dy$$

FIRST DIVIDE
THEN PARTIAL FRACTIONS

$$\frac{1}{2} y^2 - 2y + \ln|y-1| + 2\ln|y+3| + C$$

$$6 \int x \tan^{-1}(x) dx$$

BY PARTS.
 $u = \tan^{-1}(x) \quad dv = x dx$

$$\frac{1}{2} x^2 \tan^{-1}(x) - \frac{1}{2} x + \frac{1}{2} \tan^{-1}(x) + C$$

$$2 \int \frac{x^3}{\sqrt{x^2+4}} dx$$

U-SUBSTITUTION OR TRIG SUBSTITUTION
 $u = x^2 + 4 \quad x = 2 \tan(\theta)$

$$\frac{1}{3} x^2 \sqrt{x^2+4} - \frac{8}{3} \sqrt{x^2+4} + C$$

$$7 \int x e^{3x+1} dx$$

BY PARTS
 $u = x \quad dv = e^{3x+1} dx$

$$\frac{1}{3} x e^{3x+1} - \frac{1}{9} e^{3x+1} + C$$

$$3 \int_0^{\pi/4} \sin^2(t) \cos^2(t) dt$$

EVEN POWERS \Rightarrow HALF-ANGLE
(ONCE ON $\sin^2(t)$, ONCE ON $\cos^2(t)$
AND ONE MORE TIME AFTER THAT)

$$\frac{1}{8} t - \frac{1}{32} \sin(4t) + C$$

$$8 \int \frac{x^3+2}{x^2-1} dx$$

DIVIDE FIRST
THEN PARTIAL FRACTIONS

$$\frac{1}{2} x^2 + \frac{3}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + C$$

$$4 \int \sin^3(x) \tan^2(x) \cos^2(x) dx$$

MAKE ALL INTO \sin/\cos

$$\Rightarrow \int \sin^5(x) dx$$

$$\text{ODD POWER} \Rightarrow \int (1 - \cos^2(x))^2 \sin(x) dx$$

U-SUB.

$$-\cos(x) + \frac{2}{3} \cos^3(x) - \frac{1}{5} \cos^5(x) + C$$

$$9 \int_0^1 \frac{x^3}{x^2+1} dx$$

DIVIDE, FIRST

$$\int x - \frac{x}{x^2+1} dx$$

SUBSTITUTION

$$x^2+1 \left| \frac{x}{x^2+1} \right. \\ \left. \frac{x^2+x}{-x} \right.$$

$$\frac{1}{2} x^2 - \frac{1}{2} \ln|x^2+1| + C$$

$$5 \int \frac{dx}{\sqrt{4x^2+8x-12}}$$

COMPLETE SQUARE

$$\Rightarrow \sqrt{4(x^2+2x-3)} = 2\sqrt{x^2+2x+1-4} \\ = 2\sqrt{(x+1)^2-4}$$

TRIG SUB. $\Rightarrow x+1 = 2 \sec \theta$

$$\frac{1}{2} \ln \left| \frac{x+1}{2} + \frac{\sqrt{x^2+2x-3}}{2} \right| + C$$

$$10 \int \frac{1}{x^2 \sqrt{x^2-1}} dx$$

TRIG SUBSTITUTION. $x = \sec(\theta)$

$$\frac{\sqrt{x^2-1}}{x} + C$$

$$11 \int \frac{4}{x^2(x+2)} dx$$

PARTIAL FRACTIONS

$$-\frac{1}{4} \ln|x| - \frac{1}{2} \frac{1}{x} + \frac{1}{4} \ln|x+2| + C$$

$$12 \int \frac{x^3}{x^2-4} dx$$

DIVIDE, THEN PARTIAL FRACTIONS

$$\frac{1}{2} x^2 + 2 \ln|x+2| + 2 \ln|x-2| + C$$

$$13 \int \frac{x^3}{\sqrt{9-x^2}} dx$$

U-SUB OR TRIG. SUB.
 $u = 9-x^2$ $x = 3\sin(\theta)$

$$-\frac{1}{9} x^2 \sqrt{9-x^2} - 6 \sqrt{9-x^2} + C$$

$$14 \int \frac{3}{x-2\sqrt{x}} dx$$

U-SUB. $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} dx \Rightarrow dx = 2\sqrt{x} du = 2u du$

$$\int \frac{6u}{u^2-2u} du = \int \frac{6}{u-2} du$$

$$6 \ln|\sqrt{x}-2| + C$$

$$15 \int \frac{\cos(x)}{4-\sin^2(x)} dx$$

U-SUB. $u = \sin(x) \Rightarrow \int \frac{1}{4-u^2} du$

THEN PARTIAL FRACTIONS

$$\frac{1}{4} \ln|\sin(x)+2| - \frac{1}{4} \ln|\sin(x)-2| + C$$

$$16 \int \frac{e^{1/x}}{x^2} dx$$

U-SUB. $u = \frac{1}{x} \quad du = -\frac{1}{x^2} dx$

$$-\int e^u du = -e^u + C$$

$$-e^{1/x} + C$$

$$17 \int \frac{x}{x^2+2x+5} dx$$

IRREDUCIBLE \Rightarrow COMPLETE THE SQUARE

$$x^2+2x+5 = x^2+2x+1-1+5 = (x+1)^2+4$$

$$t = x+1, dt = dx \Rightarrow \int \frac{t-1}{t^2+4} dt$$

SPLIT INTO TWO.

$$\frac{1}{2} \ln|x^2+2x+5| - \frac{1}{2} \tan^{-1}\left(\frac{x+1}{2}\right) + C$$

$$18 \int \frac{1}{(9-x^2)^{3/2}} dx$$

TRIG. SUBSTITUTION. $x = 3\sin(\theta)$

$$\frac{1}{9} \frac{x}{\sqrt{9-x^2}} + C$$

$$19 \int \cos^6(3x) \sec^3(3x) dx$$

REWRITE AS ALL COSINES

$\int \cos^3(3x) dx \Rightarrow$ ODD COSINE

$$\int (1-\sin^2(3x)) \cos(3x) dx \quad \begin{matrix} u = \sin(3x) \\ du = 3\cos(3x) dx \end{matrix}$$

$$\frac{1}{3} \int (1-u^2) du$$

$$\frac{1}{3} \sin(3x) - \frac{1}{9} \sin^3(3x) + C$$

$$20 \int x^5 \ln(x) dx$$

BY PARTS $u = \ln(x) \quad du = \frac{1}{x} dx$

$$\frac{1}{6} x^6 \ln|x| - \frac{1}{36} x^6 + C$$

21 $\int \sin^3(x) \cos^6(x) dx$
 ODD SINE $\Rightarrow \int (1 - \cos^2(x)) \cos^6(x) \sin(x) dx$
 $u = \cos(x)$

$$-\frac{1}{7} \cos^7(x) + \frac{1}{9} \cos^9(x) + C$$

22 $\int \frac{5}{x^3 + 2x} dx$

PARTIAL FRACTIONS

$$\frac{5}{2} \ln|x| - \frac{5}{4} \ln|x^2 + 2| + C$$

23 $\int \frac{dx}{x^2 - 8x + 18}$

IRREDUCIBLE \Rightarrow COMPLETE THE SQUARE

$$x^2 - 8x + 18 = x^2 - 8x + 16 - 16 + 18 = (x-4)^2 + 2$$

$$u = x - 4 \quad du = dx$$

$$\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x-4}{\sqrt{2}}\right) + C$$

24 $\int \frac{dx}{x^3 + 3x^2}$

PARTIAL FRACTIONS

$$-\frac{1}{9} \ln|x| - \frac{1}{3} \frac{1}{x} + \frac{1}{9} \ln|x+3| + C$$

25 $\int \frac{e^{3x}}{e^{2x} - 1} dx$

U-SUB. $u = e^x, du = e^x dx$

$$\int \frac{u^2}{u^2 - 1} du \Rightarrow \text{DIVIDE \& PARTIAL FRACTIONS}$$

$$e^x + \frac{1}{2} \ln|e^x - 1| - \frac{1}{2} \ln|e^x + 1| + C$$

26 $\int \frac{t^3}{\sqrt{t^2 + 4}} dt$ $u = t^2 + 4$
 or $t = 2 \tan(\theta)$

U-SUB OR TRIG SUB.
 (SAME PROBLEM AS 2)

$$\frac{1}{3} t^2 \sqrt{t^2 + 4} - \frac{8}{3} \sqrt{t^2 + 4} + C$$

27 $\int 4y \sec^2(2y) dy$

BY PARTS.

$$u = 4y \quad dv = \sec^2(2y) dy \quad \begin{matrix} t = 2y \\ dt = 2dy \\ dy = \frac{1}{2} dt \end{matrix}$$

$$du = 4dy \quad v = \frac{1}{2} \tan(2y)$$

$$2y \tan(2y) - \ln|\sec(2y)| + C$$

28 $\int_0^1 x \sqrt{1-x} dx$

U-SUB. $u = 1-x \Rightarrow x = 1-u$
 $du = -dx$

$$\int (1-u) \sqrt{u} du = \int u^{1/2} - u^{3/2} du$$

$$\frac{2}{3} (1-x)^{3/2} - \frac{2}{5} (1-x)^{5/2} + C$$

29 $\int \frac{x+1}{\sqrt{5+4x-x^2}} dx$

COMPLETE THE SQUARE \Rightarrow TRIG SUB.

$$5+4x-x^2 = 5+4-4+4x-x^2 = 9-(x-2)^2$$

$$x-2 = 3 \sin(\theta)$$

$$-\sqrt{5+4x-x^2} + 3 \sin^{-1}\left(\frac{x-2}{3}\right) + C$$

30 $\int \frac{2}{x^3 - 3x^2 + x - 3} dx$

PARTIAL FRACTIONS

$$x^3 - 3x^2 + x - 3 = x^2(x-3) + (x-3)$$

$$= (x-3)(x^2+1)$$

$$\frac{1}{10} \ln|x-3| - \frac{1}{20} \ln|x^2+1| - \frac{3}{10} \tan^{-1}(x) + C$$