Work Review

Recall: When a **constant** force is applied over a given distance, then we defined Work = Force*Distance. If the force is not constant, then we approximate the work done over small subdivisions and we use calculus to get a precise measurement of work.

PROBLEM TYPE 1: When we have a force formula.

When we know of the formula for force is given by $f(x)$ and is applied from $x = a$ to $x = b$,
then Work = $\int_a^b f(x)dx$.
We saw this with springs.

1. x = distance beyond natural length
2. $f(x) = kx$ = force to hold spring at distance x (k is the spring constant).
3. So Work to stretch from $x = a$ to $x = b$ = $\int_a^b kxdx$.

Homework Hints:

On problem 4: Label the unknown natural length as L. Then if you are 0.3 m from the wall, you would have $x = 0.3 - L$.

Second to last problem: $PV^{1.4} = k$. They give you information so you can find k. Then you have $P = k/V^{1.4}$ and they tell you that work is $\int_a^b P(V)\,dV$ (just read the problem and put in the numbers they give you, it isn't a hard problem, just a lot of words).

Last problem: Force due to gravity = $\frac{GMm}{r^2}$, so work to go from $r = a$ to $r = b$ would be $\int_a^b \frac{GMm}{r^2}\,dr$

PROBLEM TYPE 2: When the force of a subdivision has a pattern for varying distances.

Cables: Let's say a cable has density 5 lbs/foot. That means that any subdivision of the cable with width Δx feet would have a force of $5\Delta x$ lbs. The distance that subdivision gets lifted depends on x (and your labeling).

If you label the top $x = 0$ and the bottom is $x = 30$ feet, then
Dist to the top = x,
Force = $5\Delta x$,
Work = $\int_0^{30} 5x\,dx$.

You could also label the top as $x = 30$ and the bottom as $x = 0$, in which case
Dist to the top = $30 - x$,
Force = $5\Delta x$,
Work = $\int_0^{30} 5(30 - x)\,dx$

(both of these integrals give the same value).

Pumping:

1. An aquarium is full of water and it is 2 feet by 3 feet on the sides and 10 feet deep. The density of water is 62.5 lbs/ft^3. If the aquarium is full of water, how much work is done in pumping all the water to the top of the aquarium?

If we label the top $x = 0$ and the bottom $x = 10$, then
Dist to the top = x.

Now we need to find the force of a subdivision at x. A subdivision with thickness Δx will have a volume of $3 * 2 * \Delta x$ ft^3, so
Force = $62.5 \cdot (3 \cdot 2 \cdot \Delta x)$ lbs.

Thus,
Work = $\int_0^{10} 62.5 \cdot 3 \cdot 2 \cdot x \, dx$.

2. A trough is in the shape of $y = x^2$ on the side and is 4 meters deep and 10 meter long. The density of water is 9800 N/m^3. If the trough is full of water, how much work is done in pumping all the water up to one meter above the top of the trough?
Here we are really forced to label the the height we want to pump to as $y = 5$ and the top of the trough as $y = 4$.
Dist to the top = $5 - y$.

Now we need to find the force of a subdivision at y (so we are in terms of y). A subdivision with thickness Δy will have a volume of $10 * 2\sqrt{y} \cdot \Delta y$ m^3, so
Force = $9800 \cdot (10 \cdot 2\sqrt{y} \cdot \Delta y)$ Newtons.

Thus,
Work = $\int_0^4 9800 \cdot 10 \cdot 2 \cdot \sqrt{y}(5 - y)\, dy$.