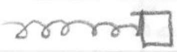
Work Examples

Springs



Spr 07 x = dist. beyond natural length
$f(x) = kx$ = force
$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i)}_{\text{FORCE}} \underbrace{\Delta x}_{\text{DIST}} = \int_a^b f(x)\,dx$$

Given: $\int_2^4 kx\,dx = 18 \Rightarrow \frac{k}{2}x^2\Big|_2^4 = 18 \Rightarrow \frac{k}{2}(4^2-2^2) = 18$
$\Rightarrow 6k = 18 \Rightarrow k = 3$

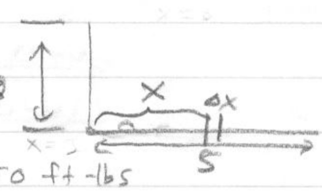
Force = 24 lbs $\Rightarrow 3x = 24 \Rightarrow \boxed{x = 8 \text{ feet}}$ ← MAX DISTANCE.

0.1 m

Spr 08 (a) x = dist. beyond natural length
FORCE $= (1\text{ kg})\cdot(9.8\text{ m/s}^2) = 9.8\text{N}$
WHEN $x = 0.03$
$\Rightarrow k(0.03) = 9.8 \Rightarrow \boxed{k = 326.\overline{6}}$

(b) $\text{work} = \int_{0.03}^{0.05} 326.\overline{6}x\,dx = \frac{326.\overline{6}}{2}x^2\Big|_{0.03}^{0.05}$
$= \frac{326.\overline{6}}{2}(0.05^2 - 0.03^2) = \boxed{0.261\overline{3} \text{ Joules}}$

Lifting Chains/Cables



Fall 06 Density $= \frac{120\text{ lbs}}{8\text{ ft}} = 15\text{ lbs/ft}$
The initial 3 ft of cable all moves up 10 feet. So work $= \underbrace{3\text{ ft}\cdot\frac{15\text{ lbs}}{\text{ft}}}_{\text{FORCE}}\cdot\underbrace{10\text{ ft}}_{\text{DIST}} = 450$ ft-lbs

If a piece of cable is x units along the ground to start then the distance traveled will be $10 - x$.
So $\text{work} = \lim_{n\to\infty}\sum_{i=1}^{n}\underbrace{15\Delta x}_{\text{FORCE}}\underbrace{(10-x_i)}_{\text{DIST}} = \int_0^5 15(10-x)\,dx = 150x - \frac{15}{2}x^2\Big|_0^5$
$= 562.5$ ft-lbs

TOTAL $= 450 + 562.5 = \boxed{1012.5 \text{ ft-lbs}}$

PUMPING

Fall 05

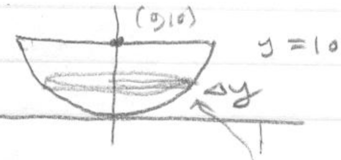
$x^2+(y-10)^2=10^2$

$\Rightarrow \quad x=\sqrt{100-(y-10)^2}$

DIST TO TOP $= 10-y_i$

FORCE = DENSITY · VOLUME OF CROSS-SECTIONAL SLICE

$= \underbrace{1000\cdot 9.8}_{\text{DENSITY } N/m^3}\cdot\underbrace{\pi\left(\sqrt{100-(y_i-10)^2}\right)^2\cdot\Delta y}_{\text{VOLUME}}$



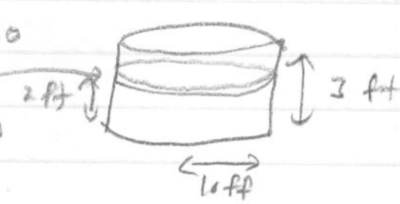
$$\boxed{\text{WORK} = \int_0^{10} 9800\pi\,(100-(y-10)^2)(10-y)\,dy}$$

Spr. 06

DIST TO TOP $= x$ ⟶ TOP $x=0$

FORCE $= \underbrace{62.5}_{\text{DENSITY}}\cdot\underbrace{\pi(10)^2\cdot\Delta x}_{\text{VOLUME}}$

BOTTOM, $x=3$



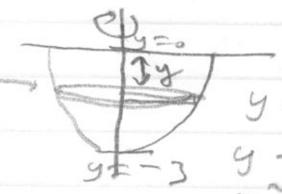
$\text{WORK} = \int_1^3 62.5\pi(10)^2\,x\,dx$

$= 6250\pi \quad \frac{1}{2}x^2\Big|_1^3 = \frac{6250}{2}\pi\,(9-1) = \boxed{25000\pi}$ ft-lbs

Win 07

DIST TO TOP $= y$

FORCE $= \underbrace{3.7\cdot 1000}_{N/m^3}\quad\underbrace{\pi\left(\sqrt{3y+9}\right)^2\Delta y}_{\text{Volume}}$



$y=\frac{1}{3}x^2-3$

$y+3=\frac{1}{3}x^2$

$x^2=3y+9$

$x=\sqrt{3y+9}$

$\text{WORK} = \int_{-3}^{0} 3700\,\pi\,(3y+9)(y)\,dy$

$= 3700\pi\int_{-3}^{0} 3y^2+9y\,dy$

$= 3700\pi\left(y^3+\frac{9}{2}y^2\Big|_{-3}^{0}\right)$

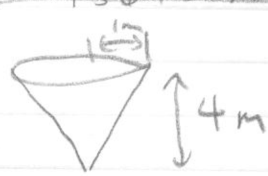
$= 3700\pi\left(0-\left((-3)^3+\frac{9}{2}(-3)^2\right)\right) = 3700\pi\left(+27-\frac{27}{2}\right)$

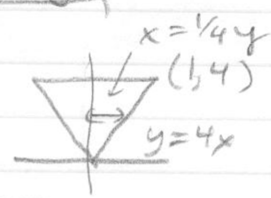
$= 3700\pi\ \ 27/2 = 49950\pi \approx 156922.55 \text{ J}$

Fall 07

DIST TO TOP $= 4-y$

FORCE $= \underbrace{9.8\cdot 1000}_{\text{DENSITY}}\quad\underbrace{\pi\left(\frac{1}{4}y\right)^2\Delta y}_{\text{VOLUME}}$





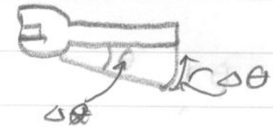
$\text{WORK} = \int_0^3 9800\pi\,\frac{1}{16}y^2(4-y)\,dy = \cdots = \boxed{\frac{77175\pi}{3} \approx 30306.6 \text{ J}}$

I'll leave this to you

OTHER

Fall 08

FORCE $= 3 + \tan^2(\theta)$ Newtons



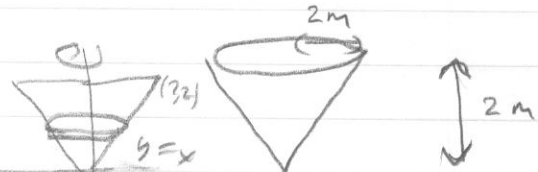
DIST $= r \cdot \Delta\theta = \underbrace{0.3}_{\text{meters}} \Delta\theta$

$$\text{WORK} = \int_0^{\pi/4} (3 + \tan^2(\theta))\, 0.3\, d\theta$$
$$= \int_0^{\pi/4} 0.9 + 0.3\tan^2(\theta)\, d\theta \qquad \tan^2\theta = \sec^2\theta - 1$$
$$= \int_0^{\pi/4} 0.9 + 0.3\sec^2\theta - 0.3\, d\theta$$
$$= 0.6\,\theta + 0.3\tan\theta \Big|_0^{\pi/4}$$
$$= (0.6\pi/4 + 0.3) - 0 = \boxed{0.15\pi + 0.3 \approx 0.77124\ \text{J}}$$

Win 09

DIST TO TOP $= 2 - y$

FORCE $= \underbrace{9.8 \cdot 1676}_{\text{DENSITY}} \cdot \underbrace{\pi (y)^2 dy}_{\text{volume}}$



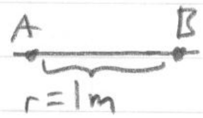
(a) $$\text{WORK} = \int_1^2 16424.8\,\pi y^2 (2-y)\, dy$$
$$= 16424.8\pi \int_1^2 2y^2 - y^3\, dy = \cdots = \boxed{47300\ \text{J}}$$

leave to you

(b) $$\text{WORK} = \int_0^1 16424.8\,\pi y^2 (2-y)\, dy = \cdots = \boxed{21500\ \text{J}}$$

Fall 09

FORCE $= -\dfrac{9\times10^9}{r^2}$



(a)
$$\text{WORK} = \int_1^2 \frac{9\times10^9}{r^2}\, dr$$
$$= 9\times10^9 \left(-\frac{1}{r}\Big|_1^2\right) = 9\times10^9\left(-\frac{1}{2} + 1\right) = \frac{9\times10^9}{2}$$
$$= \boxed{4.5\times10^9\ \text{J}}$$

FROM LATER IN THE COURSE (7.8)

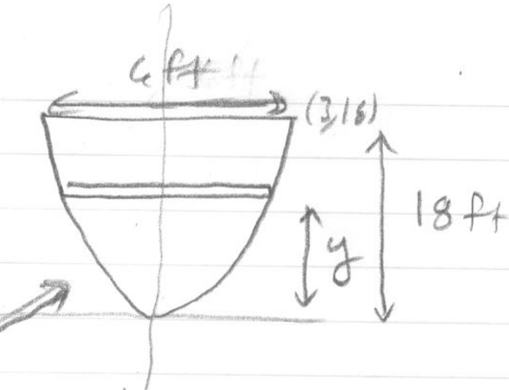
(b) $$\text{WORK} = \int_1^\infty \frac{9\times10^9}{r^2}\, dr = \lim_{t\to\infty} 9\times10^9 \int_1^t \frac{1}{r^2}\, dr$$
$$= \lim_{t\to\infty} 9\times10^9 \left[-\frac{1}{r}\Big|_1^t\right]$$
$$= \lim_{t\to\infty} 9\times10^4 \left[-\frac{1}{t} + 1\right] = \boxed{9\times10^9}\ \text{J}$$

Fall 11| $y = ax^2 + bx + c$

$b = 0$ because symmetric

$c = 0$ because it goes through the origin

$y = ax^2 \qquad 18 = a(3)^2 \Rightarrow a = 2$



$\boxed{y = 2x^2}$

$x = \sqrt{y/2}$

y	DIST moved
0	15 ft
1	16 ft
2	17 ft
y	15+y ft

DIST moved $= 15 + y$

Force $= 3 \cdot 2\sqrt{y/2} \cdot \Delta y$

$$\int_0^{18} 3 \cdot 2\sqrt{y/2}\,(15+y)\,dy$$

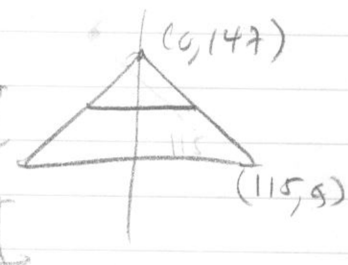
$$= \frac{6}{\sqrt{2}} \int_0^{18} 15y^{1/2} + y^{3/2}\,dy = \cdots = \boxed{\frac{27864}{5} = 5572.8 \text{ ft-lbs}}$$

Fall 12| $s(y) = 2\left(\frac{-115}{147}y + 115\right)$

DIST TO LIFT $= y$

Force $= \underset{\text{DENSITY}}{9.8 \cdot 2360}\,(s(y))^2\,\Delta y$

$\frac{147 - 0}{0 - 115}$



$\boxed{y = \frac{-147}{115}x + 147}$

$\frac{-115}{147}(y - 147) = x$

$\boxed{x = -\frac{115}{147}y + 115}$

$$\int_0^{147} 9.8 \cdot 2360 \cdot \left(2\left(\frac{-115}{147}y + 115\right)\right)^2 \cdot y\,dy = \cdots = \boxed{2.20317 \times 10^{12}\ \text{J}}$$