MATH 125 SPRING '06
SOLUTIONS

1. Evaluate the following indefinite integrals.

(a) (5 points) $\int \frac{\sin(4+\ln(y))}{y}\,dy$

$u = 4+\ln(y)$

$du = \frac{1}{y}\,dy$

$dy = y\,du$

$= \int \frac{\sin(u)}{\cancel{y}}\,\cancel{y}\,du$

$= -\cos(u) + C$

$= \boxed{-\cos(4+\ln(y)) + C}$

(b) (5 points) $\int x^3\sqrt{18-x^2}\,dx$

$u = 18-x^2 \qquad x^2 = 18-u$

$du = -2x\,dx$

$dx = \frac{du}{-2x}$

$= \int x^{\cancel{3}\,2}\sqrt{u}\,\frac{du}{-2\cancel{x}}$

$= \frac{1}{-2}\int x^2 u^{1/2}\,du$

$= -\frac{1}{2}\int (18-u)\,u^{1/2}\,du$

$= -\frac{1}{2}\int 18u^{1/2} - u^{3/2}\,du = -\frac{1}{2}\left[18\,\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right] + C$

$= \boxed{-\frac{18}{3}(18-x^2)^{3/2} + \frac{1}{5}(18-x^2)^{5/2} + C}$

2. Evaluate the following definite integrals.

(a) (5 points) $\displaystyle\int_1^e \frac{\sqrt{x}+3x}{x^2}\,dx$

$$= \int_1^e \frac{x^{1/2}}{x^2} + 3\frac{x}{x^2}\,dx$$

$$= \int_1^e x^{-3/2} + 3\frac{1}{x}\,dx$$

$$= -2x^{-1/2} + 3\ln(x)\Big|_1^e$$

$$= \left(-2e^{-1/2} + 3\ln(e)\right) - \left(-2(1)^{-1/2} + 3\ln(1)\right)$$

$$= -2e^{-1/2} + 3 - (-2)$$

$$= \boxed{-2e^{-1/2} + 5}$$

(b) (5 points) $\displaystyle\int_0^{\frac{\pi}{2}} \cos(x)\,(\sin(x))^{1/3}\,dx$

$u = \sin(x)$

$du = \cos(x)\,dx$

$dx = \frac{1}{\cos(x)}\,du$

$x = \pi/2 \Rightarrow u = 1$

$x = 0 \Rightarrow u = 0$

$$= \int_0^1 \cancel{\cos(x)}\, u^{1/3} \frac{1}{\cancel{\cos(x)}}\,du$$

$$= \frac{3}{4}u^{4/3}\Big|_0^1$$

$$= \frac{3}{4}(1)^{4/3} - \frac{3}{4}(0)^{4/3}$$

$$= \boxed{\frac{3}{4}}$$

3. A particle is moving on a straight line with acceleration given by $a(t) = -2t + 1$ and initial velocity $v(0) = 2$.

(a) (3 points) Find the velocity, $v(t)$, for the particle at time t.

$$v(t) = -\cancel{2}\tfrac{1}{\cancel{2}}t^2 + t + C = -t^2 + t + C$$

$$2 = -(0)^2 + (0) + C \Rightarrow C = 2$$

$$\boxed{v(t) = -t^2 + t + 2}$$

(b) (3 points) Find the displacement of the particle from $t = 0$ to $t = 3$.

$$\text{displacement} = \int_0^3 -t^2 + t + 2\,dt$$

$$= -\tfrac{1}{3}t^3 + \tfrac{1}{2}t^2 + 2t \Big|_0^3$$

$$= \left(-\tfrac{1}{3}3^3 + \tfrac{1}{2}3^2 + 2\cdot 3\right) - (0)$$

$$= (-9 + 4.5 + 6) = \boxed{\tfrac{3}{2} = 1.5}$$

(c) (3 points) Find the total distance traveled by the particle from $t = 0$ to $t = 3$.

$$\text{total distance} = \int_0^3 |-t^2 + t + 2|\,dt$$

$$-t^2 + t + 2 = 0 \Rightarrow t^2 - t - 2 = 0 \Rightarrow (t-2)(t+1) = 0$$

$$t = 2,\ t = -1$$

$$\int_0^2 -t^2 + t + 2\,dt - \int_2^3 -t^2 + t + 2\,dt$$

$$= \left[-\tfrac{1}{3}t^3 + \tfrac{1}{2}t^2 + 2t\Big|_0^2\right] - \left[-\tfrac{1}{3}t^3 + \tfrac{1}{2}t^2 + 2t\Big|_2^3\right]$$

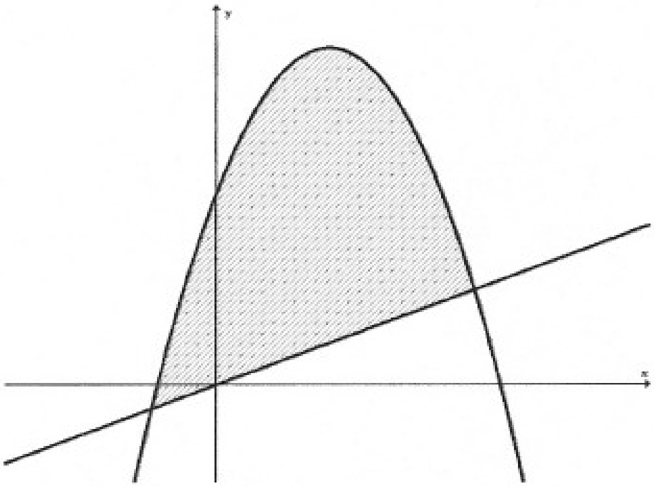
$$= \left[\left(\tfrac{10}{3}\right) - (0)\right] - \left[\left(\tfrac{3}{2}\right) - \left(\tfrac{10}{3}\right)\right] = \left[\tfrac{10}{3}\right] - \left[-\tfrac{11}{6}\right] = \boxed{\tfrac{31}{6} = 5.1\overline{6}}$$

4. (6 points)

The graph to the right illustrates the region bounded by the the two curves

$$x = 2y \quad \text{and} \quad y = -x^2 + 3.5x + 4.$$

Find the area of this region.



intersection: $y = \frac{1}{2}x$

$\frac{1}{2}x = -x^2 + 3.5x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0 \qquad x = 4,\ x = -1$

$$\text{Area} = \int_{-1}^{4} (-x^2 + 3.5x + 4) - (\tfrac{1}{2}x)\,dx$$

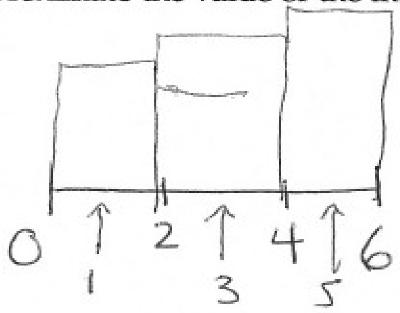
$$= \int_{-1}^{4} -x^2 + 3x + 4\,dx$$

$$= -\tfrac{1}{3}x^3 + \tfrac{3}{2}x^2 + 4x \Big|_{-1}^{4} = \left[\left(\tfrac{56}{3}\right) - \left(-\tfrac{13}{6}\right)\right] = \boxed{\frac{125}{6} = 20.8\overline{3}}$$

5. (5 points) Use the midpoint rule with $n = 3$ rectangles to approximate the value of the integral:

$$\int_0^6 \sqrt{x^3 + 1}\,dx$$

$\Delta x = \frac{6-0}{3} = 2$



$M_3 = (\text{height})\Delta x + (\text{height})\Delta x + (\text{height})\Delta x$

$= (\sqrt{1^3+1})(2) + (\sqrt{3^3+1})(2) + (\sqrt{5^3+1})(2)$

$= \boxed{\sqrt{2}\cdot 2 + \sqrt{28}\cdot 2 + \sqrt{126}\cdot 2}$

$\approx \boxed{35.8613766896 \approx 35.86}$

6. Consider the region bounded by the curves $y = x^2$ and $y = 3x$ and answer the following.

(a) (5 points) Using the method of cylindrical shells, express the volume of the solid of revolution obtained when this region is rotated around the y-axis in terms of a definite integral.
DO NOT EVALUATE THE INTEGRAL.

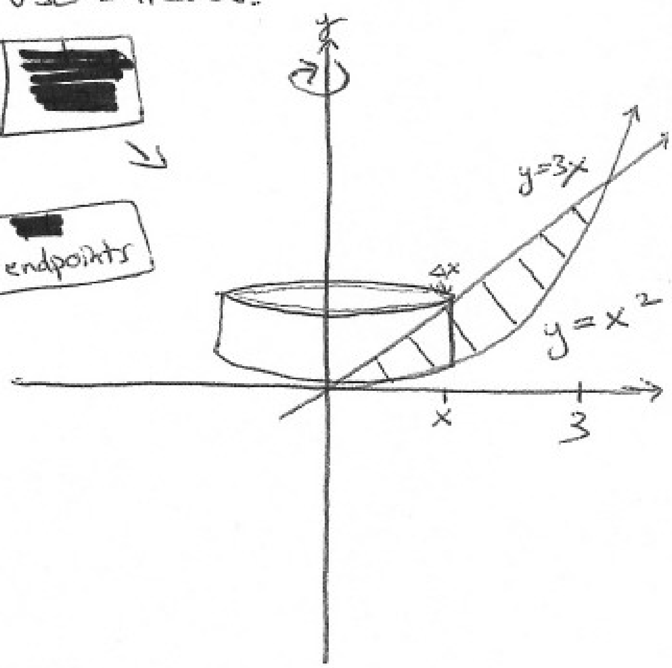
MUST USE SHELLS!

Volume $= \int_a^b 2\pi(\text{radius})(\text{height})(\text{thickness})$ ← [redacted]

intersection: $x^2 = 3x \Rightarrow x^2 - 3x = 0$
$\Rightarrow x(x-3) = 0$ ← [redacted] endpoints
$x = 0,\ x = 3$

$$\boxed{\int_0^3 2\pi x\,(3x - x^2)\,dx}$$

[redacted] radius & height



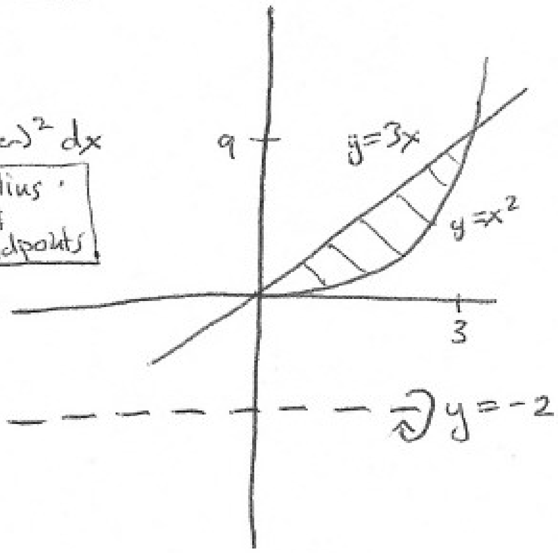
(b) (5 points) Express the volume of the solid of revolution obtained when this region is rotated around the horizontal line $y = -2$ in terms of a definite integral.
DO NOT EVALUATE THE INTEGRAL. Either Method

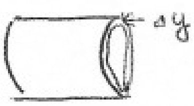
Slicing  Δx washer [redacted] setup

Volume $= \int_a^b (\text{Area of slice})(\text{thickness}) = \int_0^3 \pi(\text{outer})^2 - \pi(\text{inner})^2\,dx$

$$= \boxed{\int_0^3 \pi(3x+2)^2 - \pi(x^2+2)^2\,dx}$$

[redacted] radius & endpoints



Shells  Δy [redacted] setup

Volume $= \int_a^b 2\pi(\text{radius})(\text{height})(\text{thickness})$

$$= \boxed{\int_0^9 2\pi(y+2)\left(\sqrt{y} - \tfrac{1}{3}y\right)dy}$$

[redacted] radius height