1

REMEMBER YOUR INVERSE TRIG. DERIVATIVES (page 233)

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

1 $\int \sin^{-1}(x)\,dx$

$u = \sin^{-1}(x)$ $\quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$

$$= x\sin^{-1}(x) - \underbrace{\int \frac{x}{\sqrt{1-x^2}}\,dx}_{\text{we can do this using substitution}}$$

$u = 1-x^2$

$du = -2x\,dx$

$dx = \frac{1}{-2x}\,du$

$$= x\sin^{-1}(x) - \int \frac{\cancel{x}}{\sqrt{u}}\,\frac{1}{-2\cancel{x}}\,du$$

$$= x\sin^{-1}(x) + \frac{1}{2}\int u^{-1/2}\,du$$

$$= x\sin^{-1}(x) + \frac{1}{2}\,2u^{1/2} + C$$

$$= \boxed{x\sin^{-1}(x) + (1-x^2)^{1/2} + C}$$

2 $\int \tan^{-1}(x)\,dx$

$u = \tan^{-1}(x)$ $\quad dv = dx$

$du = \frac{1}{1+x^2}\,dx$ $\quad v = x$

$= x\tan^{-1}(x) - \underbrace{\int \frac{x}{1+x^2}\,dx}_{\text{use substitution}}$

$u = 1+x^2$

$du = 2x\,dx$

$dx = \frac{1}{2x}\,du$

$= x\tan^{-1}(x) - \int \frac{\cancel{x}}{u}\,\frac{1}{2\cancel{x}}\,du$

$= x\tan^{-1}(x) - \frac{1}{2}\int \frac{1}{u}\,du$

$= x\tan^{-1}(x) - \frac{1}{2}\ln|u| + C$

$= \boxed{x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C}$

3 $\int x\ln(x)\,dx$

$u = \ln(x)$ $\quad dv = x\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2\ln(x) - \int \frac{1}{2}x^2\frac{1}{x}\,dx$

$= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\,\frac{1}{2}x^2 + C$

$= \boxed{\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C}$

4 $\int \frac{\ln(x)}{x^2}\,dx$

$u = \ln(x)$ $\quad dv = \frac{1}{x^2}\,dx$ $\quad x^{-2}$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{1}{-1}x^{-1}$

$= -x^{-1}\ln(x) - \int -x^{-1}\frac{1}{x}\,dx$

$= -x^{-1}\ln(x) + \int x^{-2}\,dx$

$= \boxed{-\frac{\ln(x)}{x} - \frac{1}{x} + C}$

5 $\int x^2 \sin(x)\,dx$

$u = x^2$ $\quad dv = \sin(x)dx$

$du = 2x\,dx$ $\quad v = -\cos(x)$

$$= -x^2\cos(x) - \int -2x\cos(x)dx$$

$$= -x^2\cos(x) + 2\underbrace{\int x\cos(x)dx}$$

$u = x$ $\quad dv = \cos(x)dx$

$du = dx$ $\quad v = \sin(x)$

$$= -x^2\cos(x) + 2\left[x\sin(x) - \int \sin(x)dx\right]$$

$$= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)dx$$

$$= \boxed{-x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C}$$

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6 $\int e^x \sin(x)\,dx$

$u = e^x$ $\quad dv = \sin(x)dx$

$du = e^x dx$ $\quad v = -\cos(x)$

$$= -e^x\cos(x) - \int -e^x\cos(x)dx$$

$$= -e^x\cos(x) + \underbrace{\int e^x\cos(x)dx}$$

$u = e^x$ $\quad dv = \cos(x)dx$

$du = e^x dx$ $\quad v = \sin(x)$

$$= -e^x\cos(x) + e^x\sin(x) - \underbrace{\int e^x\sin(x)dx}$$

THIS IS WHAT WE STARTED WITH!

So

$$\underbrace{\int e^x\sin(x)dx} = -e^x\cos(x) + e^x\sin(x) - \underbrace{\int e^x\sin(x)dx}$$

we want to solve for

$$+\int e^x\sin(x)dx \qquad\qquad +\int e^x\sin(x)dx$$

$$2\int e^x\sin(x)dx = -e^x\cos(x) + e^x\sin(x)$$

$$\boxed{\int e^x\sin(x)dx = \frac{-e^x\cos(x) + e^x\sin(x)}{2} + C}$$

7 Same technique as for 6

$\int e^x \cos(x)dx$ $\quad u = e^x \quad dv = \cos(x)dx$

$du = e^x dx \quad v = \sin(x)$

$= e^x \sin(x) - \underbrace{\int e^x \sin(x)dx}$

$u = e^x \quad dv = \sin(x)dx$

$du = e^x dx \quad v = -\cos(x)dx$

$= e^x \sin(x) - [-e^x \cos(x) - \int -e^x \cos(x)dx]$

$= e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x)dx$

So

$$\int e^x \cos(x)dx = e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x)dx$$

$$+ \int e^x \cos(x)dx \qquad\qquad + \int e^x \cos(x)dx$$

$$2\int e^x \cos(x)dx = e^x \sin(x) + e^x \cos(x)$$

$$\boxed{\int e^x \cos(x)dx = \frac{e^x \sin(x) + e^x \cos(x)}{2} + C}$$