Chapter 6 Review

As always, my reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections. Please inform me if you find any typos on this sheet.

WARNING: Chapter 6 involves techniques for the application of integrals to areas and volumes. Although the techniques are not difficult after a little practice, it is difficult to explain and illustrate all situations using only a few sentences on a review sheet. I have tried to use common language to describe the techniques that we explore in class and in the homework. However, without carefully reviewing the lecture notes and working through homework problems, this review may be difficult to follow.

1. Areas Between Curves
   - Given a region bounded by two curves, you should know how to find the area. If you are not given the picture, you need to know techniques for figuring out (1) where the curves intersect, (2) whether to use x or y, and (3) which function is on 'top' (or to the 'right').
     - If the functions are easily expressed as $y =$ "an expression only involving x", then it is probably easiest to integrate with respect to x.

       $$\text{“Area between from } x = a \text{ to } x = b\text{”} = \int_a^b \text{TOP FUNC.} - \text{BOTTOM FUNC. } dx$$

     - If the functions are easily expressed as $x =$ "an expression only involving y", then it is probably easiest to integrate with respect to y.

       $$\text{“Area between from } y = c \text{ to } y = d\text{”} = \int_c^d \text{RIGHT FUNC.} - \text{LEFT FUNC. } dy$$

     - There are two choices using x's or using y's. If you are unsure, here is how you can try each:
       (a) Draw two pictures: (1) Draw a 'typical vertical rectangle' with small width on the side closest to the x-axis, and (2) Draw a 'typical horizontal rectangle' with small width on the side closest to the y-axis.
       (b) Choose the picture that always has the height of the rectangles bounded by the same two curves.
         * If you choose (1), use dx and write the equations and endpoints in terms of x. Write the integral as 'top function' - 'bottom function'.
         * If you choose (2), use dy and write the equations and endpoints in terms of y. Write the integral as 'right function' - 'left function'.

2. Volumes by the Slicing Method
   - Be able to effectively find volumes using the method of slicing. (Usually we apply this methods to volumes of revolution).
     (a) Draw a picture and imagine that you slice the volume across the axis of revolution. (If you slice across the x-axis, then the width corresponds to dx. If you slice across the y-axis, then the width corresponds to dy. This means use x, or y, respectively throughout the rest of the problem.)
     (b) Visualize the cross-section and write a formula for the area of the cross section.
       - If the cross-section is a circle:

         $$\text{‘Area of Slice’} = \pi(\text{radius function})^2$$

- If the cross-section is a washer:

$$\text{‘Area of Slice’} = \pi(\text{outer radius function})^2 - \pi(\text{inner radius function})^2$$

(c) Evaluate the integral using dx or dy depending on the your first step:

$$\text{Volume} = \int_a^b \text{‘Area of Slice’}$$

3. Volumes by the Shells Method

- Be able to effectively find volumes using the method of cylindrical shells.
  (a) Draw a picture and imagine that you draw a ‘pop can’ with thin outer shell around the axis of revolution. (If the width of the outer shell corresponds to x, use dx, this also is when you rotate around the y-axis. If the width of the outer shell corresponds to y, use dy, this also is when you rotate around the x-axis.)
  (b) Visualize the cylindrical shell and write a formula for the height and the circumference.
    - The height function is simply the ‘top function’ - ‘bottom function’, or ‘right function’ - ’left function’, depending on whether you are using x or y.
    - The circumference is 2π(radius formula). The radius formula is the formula for the distance from the axis of rotation to the edge of the shell. This is often of the form $x - h$, or $y - k$, where $x = h$, or $y = k$, is the axis of rotation. (In fact, in many problems the radius is just x, or y, because we rotate around an axis.)
  (c) Evaluate the integral using dx or dy depending on the your first step:

$$\text{Volume} = \int_a^b 2\pi\ (\text{‘radius formula’})(\text{‘height formula’})$$

4. Miscellaneous

- These sections can seem a little overwhelming at first. But truly there are only a few simply principles at work. Although it may seem easier initially to memorize each part of each techniques separately, it is far better for now and for later in this class that you understand how to reason through a problem by using ‘typical approximating rectangles’, ‘typical approximating slices’, and ‘typical approximating shells’. If you can comfortable draw and visualize all of these, then you should be able to easily read off all the information you need including whether to use dx or dy, what the area/radius/height formulas are, and which endpoints to use.
  - For instance, when I need to evaluate a volume, I don’t have all the techniques memorized and I don’t like to waste my time trying to look through textbooks for an example to guide me. What I typically do, is to draw a picture and re-derive the basic ideas. For example, if I wanted to use shells I would write:

$$\text{Volume} = \int_a^b (\text{‘circumference’})(\text{’height’})$$

  Then I would let my picture guide me to the end of the problem.
- Even so, I find that some students struggle with these ideas initially. As a check on your work you can remember the following (this tables only tells you which variable to use if you are using the designated method and using either the x or y-axis to rotate):

	Axis of Rotation	Axis of Rotation
Method	x-axis	y-axis
Slicing	$\text{Volume} = \int_a^b A(x)dx$	$\text{Volume} = \int_c^d A(y)dy$
Shells	$\text{Volume} = \int_a^b 2\pi y f(y)dy$	$\text{Volume} = \int_c^d 2\pi x f(x)dx$