

Random Integration Problems

$$\textcircled{1} \int \frac{x+2}{x^2+5x} dx$$

$$\textcircled{2} \int \sin^{10}(x) \cos^3(x) dx$$

$$\textcircled{3} \int x^6 \ln(x) dx$$

$$\textcircled{4} \int \frac{x^3}{x+1} dx$$

$$\textcircled{5} \int \frac{1}{\sqrt{x^2-4}} dx$$

$$\textcircled{6} \int \frac{dx}{x^3+x}$$

$$\textcircled{7} \int x \sec(x) \tan(x) dx$$

$$\textcircled{8} \int \frac{\sqrt{\tan^{-1}(x)}}{x^2+1} dx$$

$$\textcircled{9} \int \frac{dt}{\sin^2(t) + \cos(2t)}$$

$$\textcircled{10} \int \frac{x^2}{(4-x^2)^{3/2}} dx$$

$$\textcircled{11} \int \frac{dx}{1+e^x}$$

$$\textcircled{12} \int \frac{x^2+8x-3}{x^3+3x^2} dx$$

① Partial Fractions

$$\frac{x+2}{x^2+5x} = \frac{x+2}{x(x+5)} = \frac{A}{x} + \frac{B}{x+5}$$

$$x+2 = A(x+5) + Bx$$

$$x=0 \Rightarrow 2 = A(5) \quad A = \frac{2}{5}$$

$$x=-5 \Rightarrow -3 = B(-5) \quad B = \frac{3}{5}$$

$$\int \frac{x+2}{x^2+5x} dx = \int \frac{2/5}{x} + \frac{3/5}{x+5} dx$$

$$= \frac{2}{5} \int \frac{1}{x} dx + \frac{3}{5} \int \frac{1}{x+5} dx$$

$$= \boxed{\frac{2}{5} \ln|x| + \frac{3}{5} \ln|x+5| + C}$$

② TRIG ODD COS

$$\int \sin^{10}(x) \cos^2(x) \cos(x) dx$$

$$u = \sin(x)$$

$$\int \sin^{10}(x) (1 - \sin^2(x)) \cos(x) dx$$

$$du = \cos(x) dx$$

$$\int u^{10} (1 - u^2) du$$

$$\int u^{10} - u^{12} du = \frac{1}{11} u^{11} - \frac{1}{13} u^{13} + C$$

$$= \boxed{\frac{1}{11} \sin^{11}(x) - \frac{1}{13} \sin^{13}(x) + C}$$

③ By Parts log & product

$$\frac{1}{7} x^7 \ln(x) - \frac{1}{7} \int x^7 \frac{1}{x} dx$$

$$u = \ln(x)$$

$$dv = x^6 dx$$

$$du = \frac{1}{x} dx$$

$$v = \frac{1}{7} x^7$$

$$\frac{1}{7} x^7 \ln(x) - \frac{1}{7} \int x^6 dx$$

$$\boxed{\frac{1}{7} x^7 \ln(x) - \frac{1}{49} x^7 + C}$$

④ Partial Fractions (Division)

$$\frac{x^2 - x + 1}{x+1} \sqrt{x^3}$$

$$-(x^3 + x^2)$$

$$-x^2$$

$$-(-x^2 - x)$$

$$-(x+1)$$

$$-1$$

$$\int \frac{x^3}{x+1} dx$$

$$= \int x^2 - x + 1 - \frac{1}{x+1} dx$$

$$= \boxed{\frac{1}{3} x^3 - \frac{1}{2} x^2 + x - \ln|x+1| + C}$$

⑤ Trig Sub

$$x = 2 \sec(\theta)$$

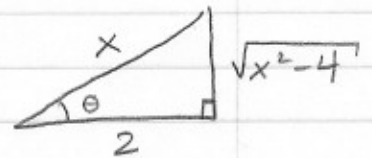
$$dx = 2 \sec(\theta) \tan(\theta) d\theta$$

$$\sqrt{x^2 - 4} = \sqrt{4 \sec^2(\theta) - 4} = 2 \tan(\theta)$$

$$\begin{aligned} \int \frac{1}{\sqrt{x^2 - 4}} dx &= \int \frac{1}{2 \tan(\theta)} \cdot 2 \sec(\theta) \tan(\theta) d\theta \\ &= \int \sec(\theta) d\theta \\ &= \ln |\sec(\theta) + \tan(\theta)| + C \\ &= \ln \left| \frac{x}{2} + \frac{\sqrt{x^2 - 4}}{2} \right| + C \end{aligned}$$

$$\boxed{\sec(\theta) = \frac{x}{2}}$$

hyp
adj



⑥ Partial Fractions

$$\frac{1}{x^3 + x} = \frac{1}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$$

$$\Rightarrow 1 = A(x^2 + 1) + (Bx + C)x$$

$$x = 0 \Rightarrow 1 = A(1) \quad \boxed{A = 1}$$

$$x = 1 \Rightarrow 1 = 2 + B + C$$

$$\boxed{-1 = B + C}$$

$$x = -1 \Rightarrow 1 = 2 - (-B + C)$$

$$\boxed{-1 = B - C} \quad B = C - 1$$

alternate method

$$\begin{cases} 1 = x^2 + 1 + Bx^2 + Cx \\ 0 = (1 + B)x^2 + Cx \\ \text{so } C = 0 \text{ or } 1 + B = 0 \end{cases}$$

$$-1 = C - 1 + C$$

$$\boxed{C = 0}$$

$$0 = 2C$$

$$\boxed{B = C - 1 = -1}$$

$$\begin{aligned} \int \frac{dx}{x^3 + x} &= \int \frac{1}{x} + \frac{-x + 0}{x^2 + 1} dx \\ &= \ln|x| - \int \frac{x}{x^2 + 1} dx \\ &= \ln|x| - \int \frac{1}{u} \cdot \frac{1}{2} du \\ &= \ln|x| - \frac{1}{2} \ln|u| + C \\ &= \ln|x| - \frac{1}{2} \ln|x^2 + 1| + C \end{aligned}$$

$$u = x^2 + 1$$

$$du = 2x dx \quad dx = \frac{1}{2x} du$$

⑦ By Parts

$$x \sec(x) - \int \sec(x) dx$$

$$\boxed{x \sec(x) - \ln |\sec(x) + \tan(x)| + C}$$

$$u = x$$

$$du = dx$$

$$dv = \sec(x) \tan(x) dx$$

$$v = \sec(x)$$

⑧ u-substitution

$$\int \frac{\sqrt{u}}{x^2+1} (x^2+1) du$$

$$= \int u^{1/2} du$$

$$= \frac{2}{3} u^{3/2} + C = \boxed{\frac{2}{3} (\tan^{-1}(x))^{3/2} + C}$$

$$u = \tan^{-1}(x)$$

$$du = \frac{1}{x^2+1} dx$$

$$dx = (x^2+1) du$$

⑨ Try identities/simplifying first

$$\sin^2(t) = \frac{1}{2}(1 - \cos(2t))$$

$$\int \frac{dt}{\sin^2(t) + \cos(2t)} = \int \frac{dt}{\frac{1}{2} - \frac{1}{2}\cos(2t) + \cos(2t)}$$

$$= \int \frac{dt}{\frac{1}{2} + \frac{1}{2}\cos(2t)}$$

$$= \int \frac{dt}{\cos^2(t)} = \int \sec^2(t) dt$$

$$= \boxed{\tan(t) + C}$$

⑩ Trig Sub

$$\int \frac{(2\sin(\theta))^2}{(4\cos^2(\theta))^{3/2}} 2\cos(\theta) d\theta$$

$$\int \frac{4\sin^2(\theta)}{(2\cos(\theta))^3} 2\cos(\theta) d\theta$$

$$\int \frac{4\sin^2(\theta)}{(2\cos(\theta))^2} d\theta = \int \frac{4\sin^2(\theta)}{4\cos^2(\theta)} d\theta = \int \tan^2(\theta) d\theta$$

$$= \int \sec^2(\theta) - 1 d\theta$$

$$= \tan(\theta) - \theta + C$$

$$= \boxed{\frac{x}{\sqrt{4-x^2}} - \sin^{-1}\left(\frac{x}{2}\right) + C}$$

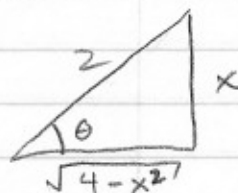
$$x = 2\sin(\theta)$$

$$dx = 2\cos(\theta) d\theta$$

$$4-x^2 = 4-4\sin^2(\theta)$$

$$= 4\cos^2(\theta)$$

$$\sin(\theta) = \frac{x}{2}$$



⑪ Simplify / Try Sub

$$\int \frac{dx}{1+e^x} = \int \frac{1}{1+u} \frac{1}{e^x} du$$

$$= \int \frac{1}{(1+u)u} du$$

$$\frac{1}{(1+u)u} = \frac{A}{u+1} + \frac{B}{u}$$

$$1 = A(u+1) + B(u)$$

$$u=0 \Rightarrow B=1 \quad u=-1 \Rightarrow A=-1$$

$$\int \frac{-1}{u+1} + \frac{1}{u} du = -\ln|u+1| + \ln|u| + C$$

$$= \boxed{-\ln|e^x+1| + \ln|e^x| + C}$$

$$u = e^x$$

$$du = e^x dx$$

$$dx = \frac{1}{e^x} du$$

(12) Partial Fractions

$$\int \frac{x^2+8x-3}{x^3+3x^2} dx = \int \frac{x^2+8x-3}{x^2(x+3)} dx$$

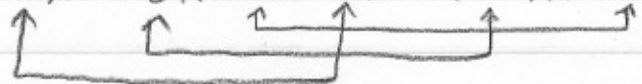
$$\frac{x^2+8x-3}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

$$x^2+8x-3 = Ax + B(x+3) + Cx^2$$

$$x=0 \Rightarrow -3 = B(3) \quad \boxed{B=-1}$$

$$x=x^2+8x-3 = Ax - x - 3 + Cx^2$$

$$x^2+8x-3 = Cx^2 + (A-1)x - 3$$



matching coefficients

$$\boxed{C=1}$$
$$\begin{cases} A+B=8 \\ 3B=-3 \end{cases}$$

$$\boxed{C=1}$$

$$A-1=8$$

$$\boxed{A=9}$$

$$\int \frac{9}{x} - \frac{1}{x^2} + \frac{1}{x+3} dx = \boxed{9 \ln|x| + x^{-1} + \ln|x+3| + C}$$