

1. (11 pts) Evaluate the integrals. If you do a substitution, you *must* show that you know how to appropriately change bounds for full credit.

$$(a) \int \frac{4}{\sqrt{x}} + x^4 \left(\frac{3}{4x^5} - \frac{x^2}{2} \right) + \sec^2(5x) dx$$

$$= \int 4x^{-1/2} + \frac{3}{4} \frac{1}{x} - \frac{1}{2} x^6 dx + \int \sec^2(5x) dx$$

$$= 8x^{1/2} + \frac{3}{4} \ln|x| - \frac{1}{14} x^7 + \int \sec^2(u) \frac{1}{5} du$$

$$\frac{1}{5} \tan(u) + C$$

$$u = 5x$$

$$du = 5 dx$$

$$\frac{1}{5} du = dx$$

$$= \left[8\sqrt{x} + \frac{3}{4} \ln|x| - \frac{1}{14} x^7 + \frac{1}{5} \tan(5x) + C \right]$$

$$(b) \int_{\pi/3}^{\pi/2} \sin(x) e^{6 \cos(x)} dx$$

$$= \int_3^0 \sin(x) e^u \frac{1}{-6 \sin(x)} du$$

$$= -\frac{1}{6} \int_3^0 e^u du$$

$$= \frac{1}{6} \int_0^3 e^u du$$

$$= \frac{1}{6} e^u \Big|_0^3$$

$$= \boxed{\frac{1}{6} (e^3 - 1)}$$

$$x = \frac{\pi}{2} \Rightarrow u = 0$$

$$u = 6 \cos(x) \quad x = \frac{\pi}{3} \Rightarrow u = 6 \cdot \frac{1}{2} = 3$$

$$du = -6 \sin(x) dx$$

$$\frac{1}{-6 \sin(x)} du = dx$$

2. (11 pts) Evaluate the integrals. If you do a substitution, you *must* show that you know how to appropriately change bounds for full credit.

$$(a) \int \frac{e^{8x}}{(e^{4x} + 5)^2} dx$$

$$u = e^{4x} + 5$$

$$e^{4x} = u - 5$$

$$du = 4e^{4x} dx$$

$$\frac{1}{4e^{4x}} du = dx$$

$$= \int \frac{e^{8x}}{u^2} \frac{1}{4e^{4x}} du$$

$$= \frac{1}{4} \int \frac{e^{4x}}{u^2} du$$

$$= \frac{1}{4} \int \frac{u-5}{u^2} du = \frac{1}{4} \int \frac{1}{u} - 5u^{-2} du$$

$$= \frac{1}{4} (\ln|u| + 5u^{-1}) + C$$

$$= \boxed{\frac{1}{4} \left(\ln(e^{4x} + 5) + \frac{5}{e^{4x} + 5} \right) + C}$$

$$(b) \int_{-3}^1 x\sqrt{x+3} dx$$

$$u = x + 3$$

$$x=1 \Rightarrow u=4$$

$$x=-3 \Rightarrow u=0$$

$$du = dx$$

$$x = u - 3$$

$$= \int_0^4 x \sqrt{u} du$$

$$= \int_0^4 (u-3) u^{1/2} du$$

$$= \int_0^4 u^{3/2} - 3u^{1/2} du$$

$$= \frac{2}{5} u^{5/2} - 2u^{3/2} \Big|_0^4$$

$$= \left(\frac{2}{5} (4)^{5/2} - 2(4)^{3/2} \right) - (0)$$

$$= \frac{2}{5} \cdot 32 - 2 \cdot 8 = \frac{64}{5} - 16 = \frac{64-80}{5} = \boxed{\frac{-16}{5}} = -3.2$$

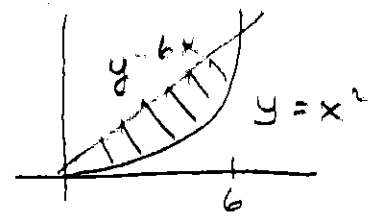
4. (12 pts)

(a) Consider the region bounded by $y = 6x$ and $y = x^2$ from $x = 0$ to $x = 6$.

i. Give the integral in terms of x for the area of this region. (DO NOT EVALUATE)

$$\int_0^6 6x - x^2 dx$$

ACTUAL
AREA
= 36



ii. Approximate the area using $n = 3$ approximating rectangles and right endpoints (don't simplify, leave your answer expanded out).

$$\Delta x = \frac{6-0}{3} = 2 \quad x_0 = 0, x_1 = 2, x_2 = 4, x_3 = 6$$

$$\left[(6(2) - (2)^2) \cdot 2 + (6(4) - (4)^2) \cdot 2 + (6(6) - (6)^2) \cdot 2 \right] = 32$$

iii. Give the pattern, in terms of i and n , for the right endpoint approximation for this area with n subdivisions.

$$\Delta x = \frac{6-0}{n}$$

$$x_i = \frac{6i}{n}$$

Answer (enter pattern here): $\sum_{i=1}^n \left[\left(6\left(\frac{6i}{n}\right) - \left(\frac{6i}{n}\right)^2 \right) \frac{6}{n} \right]$

(b) At time $t = 0$ seconds, you are standing on a bridge directly above Dr. Loveless and throw a small water balloon. It hits Dr. Loveless in 2 seconds with a downward speed of 75 feet/sec at the instant it hits him. Assume the balloon accelerates downward at a constant 32 feet/sec². At what height was the balloon exactly 1 second after you threw it?

$$a(t) = -32$$

$$v(t) = -32t + C$$

$$h(t) = -16t^2 + Ct + D$$

$$v(2) = -75 \Rightarrow -32(2) + C = -75 \Rightarrow C = -75 + 64 = -11$$

$$h(2) = 0 \Rightarrow -16(2)^2 + C(2) + D = 0 \Rightarrow -64 - 22 + D = 0 \Rightarrow D = 86$$

$$h(t) = -16t^2 - 11t + 86$$

$$h(1) = -16(1)^2 - 11(1) + 86 = -27 + 86 = 59 \text{ feet}$$

3. (12 pts)

(a) Let $f(x) = \int_{5x+\cos(x)}^{32} \frac{4}{\sqrt{8+t}} dt$. Find $f'(0)$.

$$f'(x) = -\frac{4}{\sqrt{8+5x+\cos(x)}} (5 - \sin(x)) \Rightarrow f'(0) = -\frac{4}{\sqrt{8+1}} (5-0)$$

$$= \boxed{-\frac{20}{3}}$$

(b) At t seconds, a particle is moving on a line with velocity $h'(t) = v(t) = 8t(t^2 - 1)$ ft/sec. At time $t = 2$ seconds, the particle is at a height of 10 ft (i.e. $h(2) = 10$).

i. Find the function, $h(t)$, for the height of the particle at time t seconds.

$$h(t) = \int 8t(t^2 - 1)^3 dt$$

$$= \int 8t u^3 \frac{1}{2t} du$$

$$= 4 \frac{1}{4} u^4 + C$$

$$h(t) = (t^2 - 1)^4 + C$$

$$h(2) = 10 \Rightarrow (2^2 - 1)^4 + C = 10 \Rightarrow 81 + C = 10$$

$$C = -71$$

$$\boxed{h(t) = (t^2 - 1)^4 - 71}$$

ii. Find the **total distance** traveled by the particle from $t = 0$ to $t = 2$ seconds.

$$\int_0^2 |v(t)| dt = \int_0^2 |8t(t^2 - 1)^3| dt$$

$$8t(t^2 - 1)^3 = 0 \Rightarrow t = 0, t = -1, \text{ or } t = 1$$

$$\int_0^1 8t(t^2 - 1)^3 dt = (t^2 - 1)^4 \Big|_0^1 = (0 - (-1)^4) = -1$$

$$\int_1^2 8t(t^2 - 1)^3 dt = (t^2 - 1)^4 \Big|_1^2 = 81 - 0 = 81$$

$$\text{TOTAL DISTANCE} = |-1| + 81 = \boxed{82 \text{ ft}}$$

5. (12 pts) Let R be the region bounded by $y = x^3$, the horizontal line $y = 8$, and the y -axis.

(a) Find the value of the constant b such that the horizontal line $y = b$ would divide the region R into two regions of equal area.

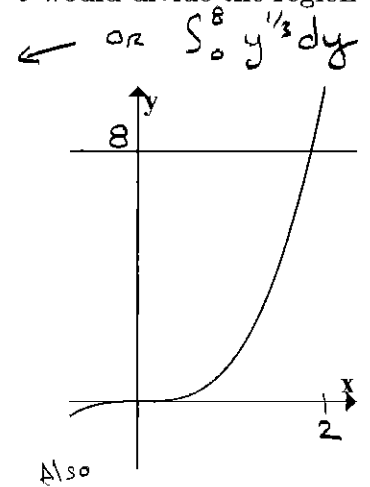
$$\text{TOTAL AREA} = \int_0^2 8 - x^3 dx = 8x - \frac{1}{4}x^4 \Big|_0^2 = 16 - 4 = 12$$

$$\text{WANT } \int_0^b y^{1/3} dy \stackrel{?}{=} \frac{1}{2}(12)$$

$$\Rightarrow \frac{3}{4} y^{4/3} \Big|_0^b = \frac{3}{4} b^{4/3} \stackrel{?}{=} 6$$

$$\Rightarrow b^{4/3} = 8$$

$$\Rightarrow \boxed{b = 8^{3/4}} \approx 4.7568$$



Also $\frac{8}{\sqrt[4]{8}} = \frac{8^{\sqrt{2}}}{\sqrt[4]{16}} = 4\sqrt{2}$

(b) Find the volume of the solid obtained by rotating the region R about the y -axis. Set up AND evaluate. (Use any correct method)

$$\int_0^8 \pi (y^{1/2})^2 dy = \pi \frac{3}{5} y^{5/3} \Big|_0^8$$

$$= \frac{3\pi}{5} 8^{5/3}$$

$$= \frac{3\pi}{5} \cdot 32$$

$$= \boxed{\frac{96\pi}{5}} \approx 60.319$$

$$\text{or } \int_0^2 2\pi x (8 - x^3) dx$$

(c) A solid is obtained by rotating the region R around the horizontal line $y = -1$. Set up the integrals you get for the volume of this solid using BOTH the method of cylindrical shells and the method of washers (DO NOT EVALUATE).

Shells: $\int_0^8 2\pi (y+1) y^{1/3} dy$

Washers: $\int_0^2 \pi (8+1)^2 - \pi (x^3+1)^2 dx$

$$\approx 420.08$$