7.7 and 7.8 Review

As always, my reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections. Please inform me if you find any typos on this sheet.

1. Approximating Integrals

   - Understand how to approximate the value of an integral by using the following 5 techniques. In all of the following $n =$ 'the number of subintervals' and $\Delta x = \frac{b-a}{n}$.
     - Left Endpoint Rule:

$$\int_a^b f(x)\ dx = \Delta x\left[f(x_0) + f(x_1) + \cdots + f(x_{n-1})\right]$$

     - Right Endpoint Rule:

$$\int_a^b f(x)\ dx = \Delta x\left[f(x_1) + f(x_2) + \cdots + f(x_n)\right]$$

     - Midpoint Rule: $\overline{x}_i = \dfrac{1}{2}(x_{i-1} + x_i)$

$$\int_a^b f(x)\ dx = \Delta x\left[f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)\right]$$

     - Trapezoid Rule:

$$\int_a^b f(x)\ dx = \frac{1}{2}\Delta x\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

     - Simpson's Rule: n has to be even

$$\begin{aligned}\int_a^b f(x)\ dx \quad &= \frac{1}{3}\Delta x[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) \\ &\qquad + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]\end{aligned}$$

   - Your first step in all of the methods above is to break up the interval into n subintervals and label x_0, x_1, *etc.*
     - For example, if you want to approximate $\int_0^4 e^{-x^2}\ dx$ using $n = 8$ and one of the methods above your first steps would be:
       (a) Compute $\Delta x = \frac{b-a}{n} = \frac{4-0}{8} = 1/2$.
       (b) Label $x_0 = 0$, $x_1 = 0.5$, $x_2 = 1$, $\ldots$, $x_7 = 3.5$, and $x_8 = 4$.
       (c) Then use the formula above that the problem asks you to use.

2. Improper Integrals

   - Infinite Integrals - Be able to evaluate and determine convergence for infinite integrals using the following steps:
     (a) Rewrite the integral as a limit.
     (b) Evaluate the integral (you do nothing with the limit in this step).
     (c) Evaluate the limit.
   - Discontinuous Integrals - Be able to identity when you are working with a discontinuous integral and be able to rewrite the problem in terms of limits. Then the steps are identical to those for infinite integrals.