(1) $\int \tan^3(x)\sec(x)\,dx$ — pull out $\sec(x)\tan(x)$ to get change rest to $\sec(x)$

$\int \tan^2(x)\sec(x)\tan(x)\,dx$

$\int (\sec^2(x)-1)\sec(x)\tan(x)\,dx$ — $u=\sec(x)$

$\int u^2-1\,du$ — $du=\sec(x)\tan(x)$

$\frac{1}{3}u^3-u+C$

$\boxed{\frac{1}{3}\sec^3(x)-\sec(x)+C}$

(2) $\int x^2\ln(x)\,dx$ — by parts!

$u=\ln(x)$ $\quad dv=x^2dx$

$du=\frac{1}{x}dx$ $\quad v=\frac{1}{3}x^3$

$=\frac{1}{3}x^3\ln(x)-\int\frac{1}{3}x^2dx$

$=\boxed{\frac{1}{3}x^3\ln(x)-\frac{1}{9}x^3+C}$

(3) $\int x\sqrt{5-x^2}\,dx$ — regular substitution! (trig. sub works too)

$u=5-x^2$

$\int \cancel{x}\sqrt{u}\,\frac{1}{-2\cancel{x}}\,du$ — $du=-2x\,dx$

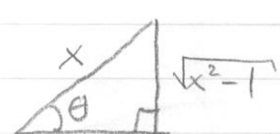
$-\frac{1}{2}\int u^{1/2}du$ — $dx=\frac{1}{-2x}du$

$-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$

$\boxed{-\frac{1}{3}(5-x^2)^{3/2}+C}$

(4) $\int \frac{\sqrt{x^2-1}}{x^2}dx$ — trig sub!

$x=\sec(\theta)$




$=\int \frac{\tan(\theta)}{\sec^2(\theta)}\sec(\theta)\tan(\theta)\,d\theta$ — $dx=\sec(\theta)\tan(\theta)$

$=\int\frac{\tan^2(\theta)}{\sec(\theta)}d\theta$

$=\int\frac{\sec^2(\theta)-1}{\sec(\theta)}d\theta=\int\sec\theta-\frac{1}{\sec\theta}\,d\theta=\int\sec(\theta)-\cos\theta\,d\theta$

$=\ln|\sec(\theta)+\tan(\theta)|-\sin(\theta)+C$

$=\boxed{\ln\left|x+\sqrt{x^2-1}\right|-\frac{\sqrt{x^2-1}}{x}+C}$

(5) $\int\frac{x^2+1}{x^2-2x-3}dx$ — divide!

$$\begin{array}{r} 1 \\ x^2-2x-3\ \overline{)\ x^2+1} \\ -(x^2-2x-3) \\ \hline 2x+4 \end{array}$$

$=\int 1+\frac{2x+4}{x^2-2x-3}dx$

$=\int 1+\frac{2x+4}{(x-3)(x+1)}dx$

$$\frac{2x+4}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$2x+4 = A(x+1) + B(x-3)$$

$$x=-1 \Rightarrow 2 = -4B \Rightarrow B = -\tfrac{1}{2}$$

$$x=3 \Rightarrow 10 = 4A \Rightarrow A = \tfrac{10}{4} = \tfrac{5}{2}$$

$$= \int 1 + \frac{5/2}{x-3} + \frac{-1/2}{x+1}\,dx$$

$$= \boxed{x + \tfrac{5}{2}\ln|x-3| - \tfrac{1}{2}\ln|x+1| + C}$$

(6) $\int y^2 (\ln(y))^2\,dy$

by parts!

$u = (\ln(y))^2 \qquad dv = y^2\,dy$

$du = 2\ln(y)\cdot\frac{1}{y}\,dy \qquad v = \frac{1}{3}y^3$

$$= \frac{1}{3}y^3(\ln(y))^2 - \int \frac{2}{3}y^2\ln(y)\,dy$$

$$= \frac{1}{3}y^3(\ln(y))^2 - \frac{2}{3}\underbrace{\int y^2\ln(y)\,dy}$$

WE DID THIS IN PROBLEM (2)

$$= \frac{1}{3}y^3(\ln(y))^2 - \frac{2}{3}\left(\frac{1}{3}y^3\ln(y) - \frac{1}{9}y^3\right) + C$$

$$= \boxed{\frac{1}{3}y^3(\ln(y))^2 - \frac{2}{9}y^3\ln(y) + \frac{2}{27}y^3 + C}$$

(7) $\int \frac{x+1}{5+4x-x^2}\,dx$

partial fractions!

$$= \int \frac{\cancel{x+1}}{(5-x)\cancel{(1+x)}}\,dx$$

CANCELLATION!

NOTE

$$= \int \frac{1}{5-x}\,dx = \boxed{-\ln|5-x| + C}$$

(8) $\int \frac{\cos(x)}{4-\sin^2(x)}\,dx$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$= \int \frac{1}{4-u^2}\,du$$

$$\frac{1}{4-u^2} = \frac{A}{2-u} + \frac{B}{2+u}$$

$$1 = A(2+u) + B(2-u)$$

$$u=2 \Rightarrow 1 = 4A \Rightarrow A = 1/4$$

$$u=-2 \Rightarrow 1 = 4B \Rightarrow B = 1/4$$

$$= \int \frac{1/4}{2-u} + \frac{1/4}{2+u}\,du$$

$$= \boxed{-\tfrac{1}{4}\ln|2-u| + \tfrac{1}{4}\ln|2+u| + C} = \boxed{-\tfrac{1}{4}\ln|2-\sin(x)| + \tfrac{1}{4}\ln|2+\sin(x)| + C}$$

(9) $\int \frac{x^2}{\sqrt{x+2}}\,dx$ $\qquad u = x+2 \qquad x = u-2$

$= \int \frac{(u-2)^2}{\sqrt{u}}\,du$ $\qquad du = dx$

$= \int \frac{u^2-4u+4}{u^{1/2}}\,du$

$= \int u^{3/2} - 4u^{1/2} + 4u^{-1/2}\,du$

$= \frac{2}{5}u^{5/2} - 4\frac{2}{3}u^{3/2} + 4\cdot 2u^{1/2} + C$

$= \boxed{\frac{2}{5}(x+2)^{5/2} - \frac{8}{3}(x+2)^{3/2} + 8(x+2)^{1/2} + C}$

(10) $\int \frac{\tan^{-1}(\sqrt{t})}{\sqrt{t}}\,dt$ $\qquad u = \sqrt{t} \iff u^2 = t$

$\int \frac{\tan^{-1}(u)}{\cancel{u}}\,2\cancel{u}\,du$ $\qquad 2u\,du = dt$

by parts!

$\int 2\tan^{-1}(u)\,du$ $\qquad u = \tan^{-1}(u) \qquad dv = 2\,du$

$du = \frac{1}{u^2+1}\,du \qquad v = 2u$

$= 2u\tan^{-1}(u) - \int \frac{2u}{u^2+1}\,du$ $\qquad w = u^2+1$

$= 2u\tan^{-1}(u) - \int \frac{1}{w}\,dw$ $\qquad dw = 2u\,du$

$= 2u\tan^{-1}(u) - \ln(w) + C$

$= \boxed{2\sqrt{t}\tan^{-1}(\sqrt{t}) - \ln(t+1) + C}$

(11) $\int \frac{\ln(\tan(\theta))}{\sin(\theta)\cos(\theta)}\,d\theta$ $\qquad t = \tan(\theta)$

$= \int \frac{\ln(t)}{\sin(\theta)\cos(\theta)}\cos^2(\theta)\,dt$ $\qquad dt = \sec^2(\theta)\,d\theta$

$d\theta = \frac{1}{\sec^2(\theta)}\,dt = \cos^2(\theta)\,dt$

$\frac{\cos(\theta)}{\sin(\theta)} = \frac{1}{\tan(\theta)} = \frac{1}{t}$

$= \int \frac{\ln(t)}{t}\,dt$ $\qquad u = \ln(t)$

$= \int u\,du$ $\qquad du = \frac{1}{t}\,dt$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln(t))^2 + C$

$= \boxed{\frac{1}{2}(\ln(\tan(\theta)))^2 + C}$