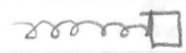
WORK EXAMPLES

SPRINGS

Spr 07 x = dist. beyond natural length 

$f(x) = kx$ = force

WORK $= \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i)}_{\text{FORCE}} \underbrace{\Delta x}_{\text{DIST}} = \int_a^b f(x)\,dx$

GIVEN: $\int_2^4 kx\,dx = 18 \Rightarrow \frac{k}{2}x^2\Big|_2^4 = 18 \Rightarrow \frac{k}{2}(4^2-2^2) = 18$

$\Rightarrow 6k = 18 \Rightarrow k=3$

FORCE = 24 lbs $\Rightarrow 3x = 24 \Rightarrow \boxed{x = 8 \text{ feet}}$ ← MAX DISTANCE.

Spr 08 (a) x = dist. beyond natural length ← 0.1m 

FORCE $= (1\ kg)\cdot(9.8\ m/s^2) = 9.8N$

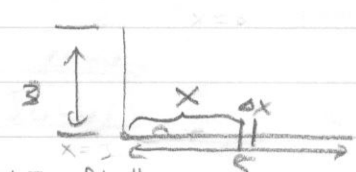
WHEN $x = 0.03$

$\Rightarrow k(0.03) = 9.8 \Rightarrow \boxed{k = 326.\overline{6}}$

(b) WORK $= \int_{0.03}^{0.05} 326.\overline{6}\,x\,dx = \frac{326.\overline{6}}{2}x^2\Big|_{0.03}^{0.05}$

$= \frac{326.\overline{6}}{2}(0.05^2 - 0.03^2) = \boxed{0.261\overline{3} \text{ Joules}}$

LIFTING CHAINS/CABLES

Fall 06 Density $= \frac{120\ lbs}{8\ ft} = 15\ lbs/ft$ 

The initial 3 ft of cable all moves up 10 feet. So work $= \underbrace{3\ ft \cdot \frac{15\ lbs}{ft}}_{\text{FORCE}} \cdot \underbrace{10\ ft}_{\text{DIST}} = 450$ ft-lbs

If a piece of cable is x units along the ground to start then the distance traveled will be $10 - x$.

So WORK $= \lim_{n\to\infty}\sum_{i=1}^{n} \underbrace{15\Delta x}_{\text{FORCE}}\underbrace{(10-x_i)}_{\text{DIST}} = \int_0^5 15(10-x)\,dx = 150x - \frac{15}{2}x^2\Big|_0^5$

$= 562.5$ ft-lbs

TOTAL $= 450 + 562.5 = \boxed{1012.5 \text{ ft-lbs}}$

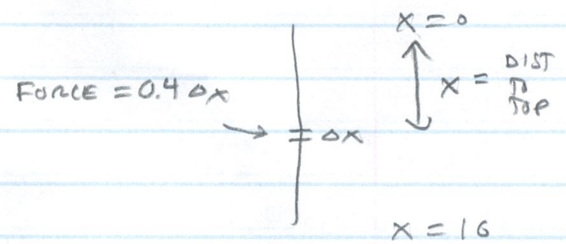
Lifting a Chain / Cables (

Spr. 10 **TRUE**

If <u>all</u> 4 pounds was lifted 10 ft then the work would be 40 ft-lbs. But not all of the rope goes up 10 feet.

Going further: Density $= \frac{4 \text{ lbs}}{10 \text{ ft}} = 0.4$ lbs/ft

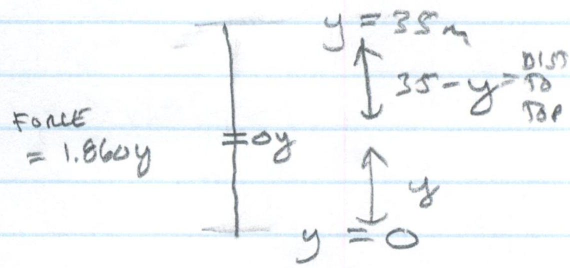
$$\text{WORK} = \int_0^{10} 0.4x\,dx = 0.2x^2\Big|_0^{10} = 0.2(10)^2 = 20 \text{ ft-lbs}$$



Fall 10 Split up the problem!

Rope DENSITY $= \frac{0.2 \text{ kg}}{\text{m}} \cdot 9.8 \text{ m/s}^2 = 1.96$ N/m

$$\int_0^{35} 1.96y\,dy = 0.98y^2\Big|_0^{35} = 0.98(35)^2 = \underbrace{1200.5 \text{ J}}_{\text{work to lift rope}}$$



Bucket ← losing weight at a constant rate ⇒ <u>LINE EQUATION</u>

20 kg · 9.8 m/s² = 196 N

2 kg · 9.8 m/s² = 19.6 N

y	FORCE (BUCKET + WATER)
0	19.6 + 196 = 215.6 N
35	215.6 − 68.6 = 147 N

$\frac{35 \text{ m}}{0.5 \text{ m/s}} = 70$ sec to get to the top

$0.1 \frac{\text{kg}}{\text{s}} \cdot 70 \text{ s} = 7$ kg will be lost due to leaking

$7 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 68.6$ N

$$\text{slope} = \frac{215.6 - 147}{0 - 35} = -1.96$$

$$\boxed{F(y) = -1.96y + 215.6}$$ ← weight of bucket at y (force) in <u>Newtons</u>

← ANSWER TO PART (a) ←

$$\text{WORK} = \int_0^{35} -1.96y + 215.6\,dy$$

$$= -0.98y^2 + 215.6y \Big|_0^{35}$$

$$= -0.98(35)^2 + 215.6(35)$$

$$= \underbrace{6345.5 \quad J}_{\text{WORK DONE TO LIFT BUCKET}}$$

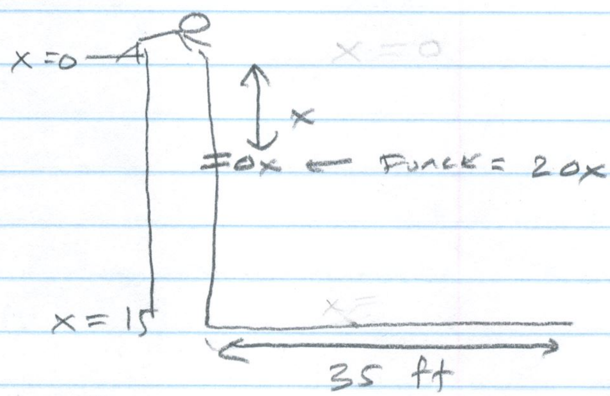
(b) TOTAL WORK $= 1200.5 + 6345.5$

$= \boxed{7546 \ J}$

Spr. 12

NOTE: ALL BITS OF ROPE ON THE GROUND WILL BE LIFTED 15 feet!

SPLIT UP THE PROBLEM!



ROPE ON GROUND

$35 \text{ ft} \cdot 2 \frac{\text{lbs}}{\text{ft}} = 70 \text{ lbs} = \text{FORCE}$

$15 \text{ ft} = \text{DISTANCE}$ ← SAME FOR ALL ROPE ON GROUND

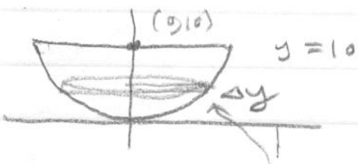
WORK $= 70 \cdot 15 = \boxed{1050 \text{ ft-lbs}}$

ROPE LIFTED UP

$$\int_0^{15} 2x\,dx = x^2 \Big|_0^{15} = (15)^2 = \boxed{225 \text{ ft-lbs}}$$

$\boxed{\text{PUMPING}}$

Fall 05 $x^2 + (y-10)^2 = 10^2$



$\Rightarrow \quad x = \sqrt{100-(y-10)^2}$

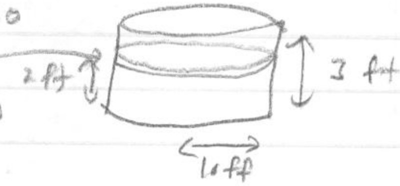
DIST TO TOP $= 10 - y_i$

FORCE = DENSITY · VOLUME OF CROSS-SECTIONAL SLICE

$$= \underbrace{1000 \cdot 9.8}_{\text{DENSITY } N/m^3} \cdot \underbrace{\pi\left(\sqrt{100-(y_i-10)^2}\right)^2}_{\text{VOLUME}} \cdot \Delta y$$

$$\boxed{\text{WORK} = \int_0^{10} 9800\pi\,(100-(y-10)^2)(10-y)\,dy}$$

Spr. 06 DIST TO TOP $= x$ ⟶ TOP $x=0$

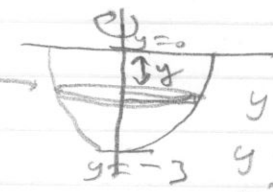


$\text{FORCE} = \underbrace{62.5}_{\text{DENSITY}} \cdot \underbrace{\pi(10)^2 \cdot \Delta x}_{\text{VOLUME}}$ BOTTOM, $x=3$

$$\text{WORK} = \int_1^3 62.5\pi(10)^2\, x\,dx$$

$$= 6250\pi \ \tfrac{1}{2}x^2\Big|_1^3 = \frac{6250}{2}\pi\,(9-1) = \boxed{25000\pi}\ \text{ft-lbs}$$

Win 07 DIST TO TOP $= y$



$\text{FORCE} = \underbrace{3.7 \cdot 1000}_{N/m^3} \ \underbrace{\pi\left(\sqrt{3y+9}\right)^2 \Delta y}_{\text{Volume}}$

$y = \frac{1}{3}x^2 - 3$

$y + 3 = \frac{1}{3}x^2$

$x^2 = 3y + 9$

$x = \sqrt{3y+9}$

$$\text{WORK} = \int_{-3}^{0} 3700\,\pi(3y+9)(y)\,dy$$

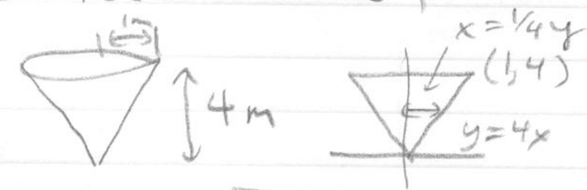
$$= 3700\pi \int_{-3}^{0} 3y^2 + 9y\,dy$$

$$= 3700\pi\left(y^3 + \tfrac{9}{2}y^2\Big|_{-3}^{0}\right)$$

$$= 3700\pi\left(0 - \left((-3)^3 + \tfrac{9}{2}(-3)^2\right)\right) = 3700\pi\left(+27 - \tfrac{27}{2}\right)$$

$$= 3700\pi\ {}^{27}\!/_2 = \boxed{49950\pi \approx 156922.55\ \text{J}}$$

Fall 07 DIST TO TOP $= 4 - y$



$\text{FORCE} = \underbrace{9.8 \cdot 1000}_{\text{DENSITY}} \ \underbrace{\pi\left(\tfrac{1}{4}y\right)^2 \Delta y}_{\text{VOLUME}}$

$$\text{WORK} = \int_0^3 9800\pi\,\tfrac{1}{16}y^2(4-y)\,dy = \cdots = \boxed{\frac{77175\pi}{3} \approx 30306.6\ \text{J}}$$

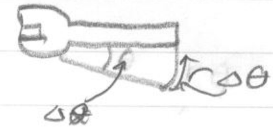
I'll leave this to you

OTHER

Fall 08

FORCE $= 3 + \tan^2(\theta)$ Newtons

DIST $= r \cdot \Delta\theta = \underbrace{0.3}_{\text{meters}} \Delta\theta$

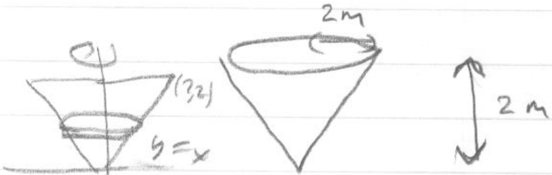


$$\text{WORK} = \int_0^{\pi/4} (3 + \tan^2(\theta))\, 0.3\, d\theta$$
$$= \int_0^{\pi/4} 0.9 + 0.3\tan^2(\theta)\, d\theta \qquad \tan^2\theta = \sec^2\theta - 1$$
$$= \int_0^{\pi/4} 0.9 + 0.3\sec^2\theta - 0.3\, d\theta$$
$$= 0.6\,\theta + 0.3\tan\theta \Big|_0^{\pi/4}$$
$$= (0.6\pi/4 + 0.3) - 0 = \boxed{0.15\pi + 0.3 \simeq 0.77124\ \text{J}}$$

Win 09

DIST TO TOP $= 2 - y$

FORCE $= \underbrace{9.8 \cdot 1676}_{\text{DENSITY}} \cdot \underbrace{\pi (y)^2\, dy}_{\text{volume}}$



(a) $\text{WORK} = \int_1^2 16424.8\, \pi y^2 (2-y)\, dy$

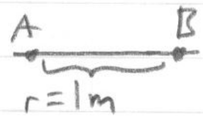
$$= 16424.8\pi \int_1^2 2y^2 - y^3\, dy = \cdots = \boxed{47300\ \text{J}}$$

leave to you

(b) $\text{WORK} = \int_0^1 16424.8\, \pi y^2 (2-y)\, dy = \cdots = \boxed{21500\ \text{J}}$

Fall 09

FORCE $= -\dfrac{9\times10^9}{r^2}$



(a)
$$\text{WORK} = \int_1^2 \frac{9\times10^9}{r^2}\, dr$$
$$= 9\times10^9 \left(-\frac{1}{r}\Big|_1^2\right) = 9\times10^9\left(-\frac{1}{2} + 1\right) = \frac{9\times10^9}{2}$$
$$= \boxed{4.5\times10^9\ \text{J}}$$

FROM LATER IN THE COURSE (7.8)

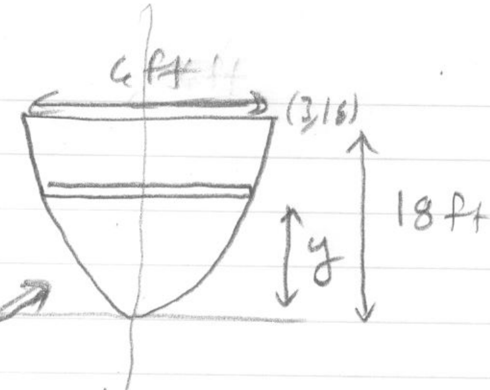
(b)
$$\text{WORK} = \int_1^\infty \frac{9\times10^9}{r^2}\, dr = \lim_{t\to\infty} 9\times10^9 \int_1^t \frac{1}{r^2}\, dr$$
$$= \lim_{t\to\infty} 9\times10^9 \left[-\frac{1}{r}\Big|_1^t\right]$$
$$= \lim_{t\to\infty} 9\times10^9 \left[-\frac{1}{t} + 1\right] = \boxed{9\times10^9}\ \text{J}$$

Fall 11 $y = ax^2 + bx + c$

$b = 0$ because symmetric

$c = 0$ because it goes through the origin

$y = ax^2 \qquad 18 = a(3)^2 \Rightarrow a = 2$



$\boxed{y = 2x^2}$

$x = \sqrt{y/2}$

DIST moved $= 15 + y$

FORCE $= 3 \cdot 2\sqrt{y/2} \cdot \Delta y$

y	DIST moved
0	15 ft
1	16 ft
2	17 ft
y	15+y ft

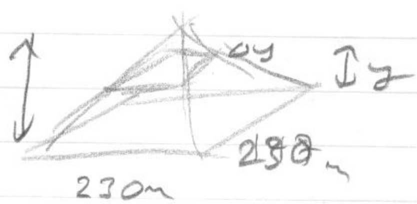
$$\int_0^{18} 3 \cdot 2\sqrt{y/2}\,(15+y)\,dy$$

$$= \frac{6}{\sqrt{2}} \int_0^{18} 15y^{1/2} + y^{3/2}\,dy = \cdots = \boxed{\frac{27864}{5} = 5572.8 \text{ ft-lbs}}$$

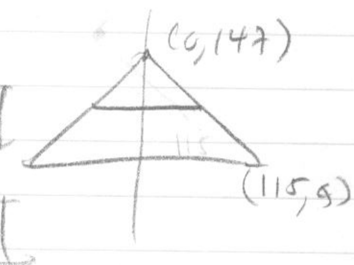
Fall 12 $s(y) = 2\left(\frac{-115}{147}y + 115\right)$

DIST TO LIFT $= y$

FORCE $= \underset{\text{DENSITY}}{9.8 \cdot 2360}\,(s(y))^2\,\Delta y$



$\frac{147 - 0}{0 - 115}$



$\boxed{y = \frac{-147}{115}x + 147}$

$\frac{-115}{147}(y - 147) = x$

$\boxed{x = -\frac{115}{147}y + 115}$

$$\int_0^{147} 9.8 \cdot 2360 \cdot \left(2\left(\frac{-115}{147}y + 115\right)\right)^2 \cdot y\,dy = \cdots = \boxed{2.20317 \times 10^{12}\ \text{J}}$$