FROM BEGINNING OF 7.2 LECTURE

Challenge integrals with solutions

① $\int \cos(x)\ln(\cos(x))\,dx$

$u=\ln(\cos(x))$ $\quad dv=\cos(x)\,dx$

$du=\frac{-\sin(x)}{\cos(x)}\,dx$ $\quad v=\sin(x)$

$$= \sin(x)\ln(\cos(x)) - \int -\frac{\sin^2(x)}{\cos(x)}\,dx \qquad \text{(by parts)}$$

$$= \sin(x)\ln(\cos(x)) + \int \frac{1-\cos^2(x)}{\cos(x)}\,dx \qquad \text{(identity)}$$

$$= \sin(x)\ln(\cos(x)) + \int \sec(x)-\cos(x)\,dx$$

$$= \boxed{\sin(x)\ln(\cos(x)) + \ln|\sec(x)+\tan(x)| - \sin(x) + C}$$

② $\int x e^x \ln(x-1)\,dx$

BY PARTS

$u=\ln(x-1)$ $\quad dv=xe^x\,dx$

$du=\frac{1}{x-1}\,dx$ $\quad v=(x-1)e^x$

$$=(x-1)e^x\ln(x-1) - \int \frac{1}{\cancel{x-1}}\cancel{(x-1)}e^x\,dx$$

$$=\boxed{(x-1)e^x\ln(x-1) - e^x + c}$$

ASIDE $\int xe^x\,dx$ $\quad u=x \quad dv=e^x dx$

$du=dx \quad v=e^x$

$= xe^x - \int e^x\,dx$

$= xe^x - e^x + c$

$=(x-1)e^x + c$

③ $\int \sin(x)\cos(x)\,e^{\cos(x)}\,dx$

$t=\cos(x)$

$dt=-\sin(x)\,dx$

$dx=-\frac{1}{\sin(x)}\,dt$

$$=\int \cancel{\sin(x)}\,t e^t \frac{1}{-\cancel{\sin(x)}}\,dt$$

$$= -\int te^t\,dt$$

$$= \boxed{-(t-1)e^t + C}, = \boxed{-(\cos(x)-1)e^{\cos(x)} + c}$$

④ $\int x^4\tan^{-1}(x)\,dx$

BY PARTS

$u=\tan^{-1}(x)$ $\quad dv=x^4\,dx$

$du=\frac{1}{1+x^2}\,dx$ $\quad v=\frac{1}{5}x^5$

$$=\frac{1}{5}x^5\tan^{-1}(x) - \frac{1}{5}\int \frac{x^5}{1+x^2}\,dx$$

$t=1+x^2 \quad x^2=t-1$

$dt=2x\,dx$

$dx=\frac{1}{2x}\,dt$

$$=\frac{1}{5}x^5\tan^{-1}(x) - \frac{1}{5}\int \frac{\cancel{x^5}^{\,4}}{t}\frac{1}{2\cancel{x}}\,dt$$

$x^4=(t-1)^2$

$$=\frac{1}{5}x^5\tan^{-1}(x) - \frac{1}{10}\int \frac{(t-1)^2}{t}\,dt$$

$$=\frac{1}{5}x^5\tan^{-1}(x) - \frac{1}{10}\int \frac{t^2-2t+1}{t}\,dt$$

$$=\frac{1}{5}x^5\tan^{-1}(x) - \frac{1}{10}\int t-2+\frac{1}{t}\,dt$$

$$=\frac{1}{5}x^5\tan^{-1}(x) - \frac{1}{20}t^2 + \frac{2}{10}t - \frac{1}{10}\ln|t| + C$$

$$=\boxed{\frac{1}{5}x^5\tan^{-1}(x) - \frac{1}{20}(1+x^2)^2 + \frac{1}{5}(1+x^2) - \frac{1}{10}\ln|1+x^2| + c}$$

NEED TO DO INTEGRATION BY PARTS **5 TIMES**.

⑤ $\int x^5 \sin(x)\,dx$

$u = x^5 \quad dv = \sin(x)dx$

$du = 4x^4 \quad v = -\cos(x)$

$= -x^5\cos(x) - \int (4x^4)(-\cos(x))dx$

$u = 4x^4 \quad dv = -\cos(x)\,dx$

$du = 16x^3 \quad v = -\sin(x)$

$= -x^5\cos(x) - [-4x^4\sin(x) - \int 16x^3(-\sin(x))dx]$

$= -x^5\cos(x) + 4x^4\sin(x) + \int (16x^3)(-\sin(x))dx$

$u = 16x^3 \quad dv = -\sin(x)dx$

$du = 48x^2dx \quad v = \cos(x)$

$= -x^5\cos(x) + 4x^4\sin(x) + 16x^3\cos(x) - \int 48x^2\cos(x)dx$

$u = 48x^2 \quad dv = \cos(x)\,dx$

$du = 96x\,dx \quad v = \sin(x)$

$= -x^5\cos(x) + 4x^4\sin(x) + 16x^3\cos(x) - [48x^2\sin(x) - \int 96x\sin(x)dx]$

$= -x^5\cos(x) + 4x^4\sin(x) + 16x^3\cos(x) - 48x^2\sin(x) + \int 96x\sin(x)dx$

$u = 96x \quad dv = \sin(x)dx$

$du = 96dx \quad v = -\cos(x)$

$= -x^5\cos(x) + 4x^4\sin(x) + 16x^3\cos(x) - 48x^2\sin(x) - 96x\cos(x) - \int 96(-\cos(x))dx$

$= \boxed{-x^5\cos(x) + 4x^4\sin(x) + 16x^3\cos(x) - 48x^2\sin(x) - 96x\cos(x) + 96\sin(x) + C}$

YOU WILL NEVER HAVE TO DO THIS MANY STEPS ON AN EXAM.

OFTEN, people save a lot of writing by doing all the derivatives and integrations from the by parts at first in a table on the right. (You can see, from how I set it up, that each u is the derivative of the previous and each v is the integral of the previous). Then keep track of alternating sign. This is called tabular integration by parts (and you don't need to know it for this class).