8.3, 9.1, 9.3 and 9.4 Review

This review sheet discusses, in a very basic way, the key concepts from these sections. This review is not meant to be all inclusive, but hopefully it reminds you of some of the basics. Please notify me if you find any typos in this review.

1. Section 8.3 (Center of Mass/Centroid): Be able to find the centroid of a flat uniformly distributed plate bounded by given functions in the plane.

   - If it is easier to determine the area in terms of x use:

   $$\overline{x} = \frac{\int_a^b xf(x)\ dx}{\int_a^b f(x)\ dx} \quad \text{and} \quad \overline{y} = \frac{\int_a^b \frac{1}{2}[f(x)]^2\ dx}{\int_a^b f(x)\ dx}$$

   - If it is easier to determine the area in terms of y use:

   $$\overline{y} = \frac{\int_c^d yf(y)\ dy}{\int_c^d f(y)\ dy} \quad \text{and} \quad \overline{x} = \frac{\int_c^d \frac{1}{2}[f(y)]^2\ dy}{\int_c^d f(y)\ dy}$$

   - For the centroid of a region bounded by two curves ($f(x) > g(x)$), use the following (this is written in terms of x):

   $$\overline{x} = \frac{\int_a^b x[f(x) - g(x)]\ dx}{\int_a^b [f(x) - g(x)]\ dx} \quad \text{and} \quad \overline{y} = \frac{\int_a^b \frac{1}{2}([f(x)]^2 - [g(x)]^2)\ dx}{\int_a^b [f(x) - g(x)]\ dx}$$

2. Section 9.1 (Intro. to Differential Equations): Understand the basic terminology for differential equations. In particular, you should know the definition of all of the following (these are loose definitions):

   - A *differential equation* is an equation involving a function and its derivatives. An *ordinary differential equation, ODE,* involves only two variables (independent and dependent) and a *partial differential equation, PDE,* involves more than two variables. We only talk about ODE's in this course.
   - An *initial condition* is a value of the function that accompanies a differential equation and allows us to find the undetermined constant in our solution. An *initial-value problem* is a differential equation with an initial condition.
   - The *order* of a differential equation is the highest derivative that appears in the equation. In this course we talk mostly about first-order differential equations.
   - A *constant solution* or *equilibrium solution* is a solution of the form $y(x) = c$ for some constant c (notice that this causes the derivative to be 0 for all x). We solve for the equilibrium solution by setting our derivatives equal to zero.
   - A *solution* to a differential equation is a function that satisfies the equation. There are typically infinitely many solutions which can be written in a general form by including an undetermined constant. To check if you have a solution, you must plug your function into the left and right-hand sides of the differential equation and see if they are equal.

   Differential equations can be used to model many natural phenomena and is a great analytic tool for testing hypotheses about natural phenomena. Although this is not a course in mathematical modeling, you should know how to interpret the physical significance of different parts of a differential equations. Perhaps the most useful thing to remember when modeling is that:

   $$\frac{dy}{dt} = \text{`the instantaneous rate of change of } y \text{ with respect to } t\text{'}$$

   So for example, the differential equation $\frac{dP}{dt} = kP$ says that the rate of change of P is proportional to the size of P.

3. Section 9.3 (Separation of Variables): This is the only technique that you will learn in this course for solving differential equations. A differential equations is a *separable equation* if you can write it in the form:

$$\frac{dy}{dx} = g(x)f(y)$$

that is, if you can separate the variables. If you can separate the variables then you can use integrals to solve the differential equation as follows:

   (a) SEPARATE VARIABLES - you may need to simply or factor to start, then use only multiplication or division to get all the x's to one side and all the y's to the other.

   (b) INTEGRATE - integrate each side separately, put the undetermined constants all on one side.

   (c) SOLVE FOR $y(x)$ (IF POSSIBLE) - to get a nice clean looking solution, we often write our solutions as a function $y(x)$. We will then replace all undetermined constants with a new constant.

   (d) USE INITIAL CONDITION - this will allow you to solve for undetermined constants.

   (e) CHECK - Always check that your solution is correct.

4. Section 9.4/3.8: Know how to interpret a differential equation in a story problem.

   (a) Know how to take a story with a given differential equation by first solving the differential equation (using 9.3 material), plugging in initial conditions, and interpreting the answer.

   (b) Know the particular examples that we spent extra energy on (these are the models that you would be expected to know from memory): Populations according to natural growth and Newton's Law of Cooling.