CHAPTER 7: ESSENTIAL TRIG IDENTITIES

We can get all the identities we need from this course from the four identities (Formulas (2), (3), and (4) can be derived from the law of cosines, see exercises 83, 85 and 86 in Appendix D if you want to learn more).

$$
\begin{aligned}
&(1)\ \sin^2(x) + \cos^2(x) = 1 \\
&(2)\ \sin(A)\cos(B) &&= \tfrac{1}{2}\left[\sin(A-B) + \sin(A+B)\right] \\
&(3)\ \cos(A)\cos(B) &&= \tfrac{1}{2}\left[\cos(A-B) + \cos(A+B)\right] \\
&(4)\ \sin(A)\sin(B) &&= \tfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]
\end{aligned}
$$

Here's how to get the other identities we need:

Dividing (1) by $\cos^2(x)$ gives:	$\tan^2(x) + 1$	$= \sec^2(x)$
Dividing (1) by $\sin^2(x)$ gives:	$1 + \cot^2(x)$	$= \csc^2(x)$
Plugging $A = B = x$ into (2) gives:	$\sin(x)\cos(x)$	$= \frac{1}{2}(\sin(2x))$
Plugging $A = B = x$ into (3) gives:	$\cos^2(x)$	$= \frac{1}{2}(1 + \cos(2x))$
Plugging $A = B = x$ into (4) gives:	$\sin^2(x)$	$= \frac{1}{2}(1 - \cos(2x))$

Plugging $A = mx$ and $B = nx$ into (2), (3) and (4) give

$$\sin(mx)\cos(nx) = \tfrac{1}{2}[\sin((m-n)x) + \sin((m+n)x)]$$

$$\cos(mx)\cos(nx) = \tfrac{1}{2}[\cos((m-n)x) + \cos((m+n)x)]$$

$$\sin(mx)\sin(nx) = \tfrac{1}{2}[\cos((m-n)x) - \cos((m+n)x)]$$