1. $\int \frac{y^3-4y+7}{y^2+2y-3}\,dy = \int y-2+\frac{3y+1}{y^2+2y-3}\,dy$

FIRST DIVIDE, THEN PARTIAL FRACTIONS

$\frac{1}{2}y^2-2y+\ln|y-1|+2\ln|y+3|+C$

2. $\int \frac{x^3}{\sqrt{x^2+4}}\,dx = \int \frac{u-4}{u^{1/2}}\,\frac{1}{2}\,du$

SUBSTITUTION WITH $u=x^2+4$

OR TRIG SUB WITH $x=2\tan\theta$

$\frac{1}{3}(x^2+4)^{3/2}-4\sqrt{x^2+4}+C$

3. $\int \sin^2(t)\cos^2(t)\,dt = \int \frac{1}{2}(1-\cos(2t))\frac{1}{2}(1+\cos(2t))\,dt$

EVEN POWERS OF SINE & COSINE! EXPAND!

HALF-ANGLE

$\frac{1}{8}t-\frac{1}{32}\sin(4t)+C$

4. $\int \sin^3(x)\tan^2(x)\cos^2(x)\,dx = \int \sin^5(x)\,dx$

MAKE ALL SINE & COSINE! ↓

NOTE $\tan(x)=\frac{\sin(x)}{\cos(x)}$. $\int (1-\cos^2(x))^2\sin(x)\,dx$

THEN ODD POWER ON SINE! $u=\cos(x)$

$-\cos(x)+\frac{2}{3}\cos^3(x)-\frac{1}{5}\cos^5(x)+C$

5. $\int \frac{dx}{\sqrt{4x^2+8x-12}} = \int \frac{dx}{2\sqrt{(x+1)^2-4}}$

COMPLETE SQUARE!

THEN TRIG. SUB. $x+1=2\sec\theta$

$\frac{1}{2}\left|\ln\left|\frac{x+1}{2}+\frac{\sqrt{x^2+2x-3}}{2}\right|\right|+C$

6. $\int x\tan^{-1}(x)\,dx = \frac{1}{2}x^2\tan^{-1}(x)-\int \frac{\frac{1}{2}x^2}{x^2+1}\,dx$

BY PARTS

$u=\tan^{-1}(x)$ $dv=x\,dx$

$du=\frac{1}{x^2+1}\,dx$ $v=\frac{1}{2}x^2$

DIVIDE!

$\frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}x+\frac{1}{2}\tan^{-1}(x)+C$

7. $\int xe^{3x+1}\,dx = \frac{1}{3}xe^{3x+1}-\int \frac{1}{3}e^{3x+1}\,dx$

BY PARTS

$u=x$ $dv=e^{3x+1}\,dx$

$du=dx$ $v=\frac{1}{3}e^{3x+1}$

$\frac{1}{3}xe^{3x+1}-\frac{1}{9}e^{3x+1}+C$

8. $\int \frac{x^3+2}{x^2-1}\,dx = \int x+\frac{-x+2}{x^2-1}\,dx$

FIRST DIVIDE, THEN PARTIAL FRACTIONS

$\frac{1}{2}x^2+\frac{3}{2}\ln|x-1|-\frac{1}{2}\ln|x+1|+C$

9. $\int \frac{x^3}{x^2+1}\,dx = \int x-\frac{x}{x^2+1}\,dx$ $u=x^2+1$

DIVIDE, THEN IRREDUCIBLE QUAD. OR START WITH A SUBSTITUTION

$\frac{1}{2}x^2-\frac{1}{2}\ln|x^2+1|+C$

10. $\int \frac{1}{x^2\sqrt{x^2-1}}\,dx = \int \frac{\sec\theta\tan\theta}{\sec^2\theta\tan\theta}\,d\theta$

TRIG. SUB. $x=\sec\theta$ $= \int \cos\theta\,d\theta$

$\frac{\sqrt{x^2-1}}{x}+C$

11. $\int \frac{4}{x^2(x+2)}\,dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}\,dx$

PARTIAL FRACTIONS!

$$-\ln|x| - \frac{2}{x} + \ln|x+2| + C$$

12. $\int \frac{x^3}{x^2-4}\,dx = \int x + \frac{4x}{x^2-4}\,dx$

DIVIDE, THEN PARTIAL FRAC.

$$\frac{1}{2}x^2 + 2\ln|x+2| + 2\ln|x-2| + C$$

13. $\int \frac{x^3}{\sqrt{9-x^2}}\,dx = \int \frac{9-u}{u^{1/2}} \frac{-1}{2}\,du$

SUB. $u = 9-x^2$

OR TRIG. SUB $x = 3\sin\theta$

$$-9\sqrt{9-x^2} + \frac{1}{3}(9-x^2)^{3/2} + C$$

14. $\int \frac{3}{x-2\sqrt{x}}\,dx = \int \frac{6t}{t^2-2t}\,dt$

SUB. $t = \sqrt{x} \Leftrightarrow t^2 = x$

$$6\ln|\sqrt{x}-2| + C$$

15. $\int \frac{\cos(x)}{4-\sin^2(x)}\,dx = \int \frac{1}{4-t^2}\,dt$

SUB. $t = \sin(x)$

THEN PARTIAL FRACTIONS!

$$\frac{1}{4}\ln|\sin(x)+2| - \frac{1}{4}\ln|\sin(x)-2| + C$$

16. $\int \frac{e^{1/x}}{x^2}\,dx = -\int e^u\,du$

SUBSTITUTION! $u = \frac{1}{x}$ $du = -\frac{1}{x^2}dx$

$$-e^{1/x} + C$$

17. $\int \frac{x}{x^2+2x+5}\,dx = \int \frac{x}{(x+1)^2+4}\,dx$

IRREDUCIBLE $\Rightarrow$ COMPLETE SQUARE

SUB: $t = x+1 \Rightarrow \int \frac{t-1}{t^2+4}\,dt$

SPLIT UP! $\underbrace{\int \frac{t}{t^2+4}\,dt}_{\text{SUB}} - \underbrace{\int \frac{1}{t^2+4}\,dt}_{\text{IN TABLE!}}$

$$\frac{1}{2}\ln|x^2+2x+5| - \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$$

18. $\int \frac{1}{(9-x^2)^{3/2}}\,dx = \int \frac{3\cos\theta}{(3\cos\theta)^3}\,d\theta$

TRIG. SUB: $x = 3\sin\theta$ $= \int \frac{1}{9}\sec^2\theta\,d\theta$

$= \frac{1}{9}\tan\theta + C$

$$\frac{1}{9}\frac{x}{\sqrt{9-x^2}} + C$$

19. $\int \cos^6(3x)\sec^3(3x)\,dx = \int \cos^3(3x)\,dx$

ODD POWER ON COSINE! $u = \sin(3x)$

$\int (1-\sin^2(3x))\cos(3x)\,dx$

$\int (1-u^2)\frac{1}{3}\,du$

$$\frac{1}{3}\sin(3x) - \frac{1}{9}\sin^3(3x) + C$$

20. $\int x^5\ln(x)\,dx = \frac{1}{6}x^6\ln(x) - \int \frac{1}{6}x^5\,dx$

BY PARTS $u = \ln(x)$ $dv = x^5dx$

$du = \frac{1}{x}dx$ $v = \frac{1}{6}x^6$

$$\frac{1}{6}x^6\ln(x) - \frac{1}{36}x^6 + C$$

21. $\int \sin^3(x)\cos^6(x)\,dx$

ODD SINE $\Rightarrow \int (1-\cos^2(x))\cos^6(x)\sin(x)$

$u = \cos(x)$

$$-\frac{1}{7}\cos^7(x) + \frac{1}{9}\cos^9(x) + C$$

22. $\int \frac{5}{x^3+2x}\,dx = \int \frac{5}{x(x^2+2)}\,dx$

PARTIAL FRACTIONS $\frac{A}{x} + \frac{Bx+C}{x^2+2}$

$$\frac{5}{2}\ln|x| - \frac{5}{4}\ln|x^2+2| + C$$

23. $\int \frac{dx}{x^2-8x+18} = \int \frac{dx}{(x-4)^2+2}$

IRREDUCIBLE

$t = x-4$ OR TRIG SUB $x-4 = \sqrt{2}\tan\theta$

$$\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-4}{\sqrt{2}}\right) + C$$

24. $\int \frac{dx}{x^3+3x^2}\,dx = \int \frac{dx}{x^2(x+3)}$

PARTIAL FRAC $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$

$$-\frac{1}{9}\ln|x| - \frac{1}{3x} + \frac{1}{9}\ln|x+3| + C$$

25. $\int \frac{e^{3x}}{e^{2x}-1}\,dx = \int \frac{t}{t^2-1}\,dt$

SUB. $t = e^x$

$$e^x + \frac{1}{2}\ln|e^x-1| - \frac{1}{2}\ln|e^x+1| + C$$

26. $\int \frac{t^3}{\sqrt{t^2+4}}\,dt$

SUBSTITUTION $u = t^2+4$

OR TRIG SUB. $t = 2\tan\theta$

$$\frac{1}{3}(t^2+4)^{3/2} - 4\sqrt{t^2+4} + C$$

27. $\int 4y\sec^2(2y)\,dy = 2y\tan(2y) - \int 2\tan(2y)\,dy$

BY PARTS! $u = 4y$, $dv = \sec^2(2y)\,dy$

$du = 4\,dy$, $v = \frac{1}{2}\tan(2y)$

$$2y\tan(2y) - \ln|\sec(2y)| + C$$

28. $\int x\sqrt{1-x}\,dx = -\int (1-t)\sqrt{t}\,dt$

SUB. $t = 1-x \rightarrow x = 1-t$

$$-\frac{2}{3}(1-x)^{3/2} + \frac{2}{5}(1-x)^{5/2} + C$$

29. $\int \frac{x+1}{\sqrt{5+4x-x^2}}\,dx = \int \frac{x+1}{\sqrt{9-(x-2)^2}}\,dx$

COMPLETE SQUARE!

TRIG SUB. $x-2 = 3\sin\theta$

$$-\sqrt{5+4x-x^2} + 3\sin^{-1}\left(\frac{x-2}{3}\right) + C$$

30. $\int \frac{1}{x^3-3x^2+x-3}\,dx = \int \frac{1}{x^2(x-3)+(x-3)}\,dx$

PART. FRAC. $= \int \frac{1}{(x-3)(x^2+1)}\,dx$

$\frac{A}{x-3} + \frac{Bx+C}{x^2+1}$

$$\frac{1}{10}\ln|x-3| - \frac{1}{20}\ln|x^2+1| - \frac{3}{10}\tan^{-1}(x) + C$$