CHAPTER 7: TRIG INTEGRALS

1. SINES AND COSINES

(a) If \( \sin(x) \) or \( \cos(x) \) is to an odd power.
   
i. Factor out a term from the odd power.
   
ii. Use the identity \( \sin^2(x) + \cos^2(x) = 1 \).
   
iii. Do a substitution \( u = \sin(x) \) or \( u = \cos(x) \) as appropriate).

(b) If \( \sin(x) \) and \( \cos(x) \) both have even powers.
   
i. Simplify with half-angle identities

2. TANGENTS AND SECANTS

(a) If \( \sec(x) \) has an even power.
   
i. Factor out \( \sec^2(x) \).
   
ii. Use the identity \( \sec^2(x) = \tan^2(x) + 1 \).
   
iii. Do a substitution \( u = \tan(x) \).

(b) If \( \tan(x) \) has an odd power.
   
i. Factor out \( \sec(x) \tan(x) \).
   
ii. Use the identity \( \tan^2(x) = \sec^2(x) - 1 \).
   
iii. Do a substitution \( u = \sec(x) \).

3. NOTES

(a) For \( \cot(x)/\csc(x) \) the cases would nearly identical to \( \tan(x)/\sec(x) \).

(b) If you are given an integral that contains \( \sin(x)/\cos(x) \) along with \( \sec(x)/\tan(x) \), it is typically best to first change everything into \( \sin(x)/\cos(x) \) (or change everything into \( \sec(x)/\tan(x) \)).

(c) Remember that we have added the following to our table of known integrals:

\[
\int \tan(x) \, dx = \ln |\sec(x)| + C \quad \text{(in 5.5)}
\]

\[
\int \sec(x) \, dx = \ln |\sec(x) + \tan(x)| + C \quad \text{(in 7.2)}
\]

\[
\int \sec^3(x) \, dx = \frac{1}{2} (\sec(x) \tan(x) + \ln |\sec(x) + \tan(x)|) + C \quad \text{(in 7.2)}
\]