Math 125
Exam 2
February 23, 2023

Name _____

Student ID #_____   Section _____

- This exam consists of a cover, a scratch sheet, five pages of questions, and another scratch sheet. If you put work on either scratch sheet and you want it to be graded, then you must clearly tell us in the problem to "see first scratch page" or "see last scratch page".

- Turn off and stow away all cell phones, smart watches, music players, and other similar devices.

- You may use one $8.5'' \times 11''$ sheet of handwritten notes. You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.

- You must **show your work**. You will NOT get credit if there is no or incomplete work, even if your final answer is correct.

- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.

- *Special Note for Trig Sub*: An answer containing an inverse trig inside of a trig function, such as $\cos(\sin^{-1}(x))$ or $\sin(2\cos^{-1}(x))$, is NOT acceptable, instead show that you can simplify using the triangle method from class.

- You may use directly the integral formulas in the table below. You must **show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas   Constants of integration have been omitted.

1. $\int x^n dx = \dfrac{x^{n+1}}{n+1}$ $(n \neq -1)$	**2.** $\int \dfrac{1}{x} dx = \ln|x|$
3. $\int e^x dx = e^x$	**4.** $\int b^x dx = \dfrac{b^x}{\ln b}$
5. $\int \sin x\, dx = -\cos x$	**6.** $\int \cos x\, dx = \sin x$
7. $\int \sec^2 x\, dx = \tan x$	**8.** $\int \csc^2 x\, dx = -\cot x$
9. $\int \sec x \tan x\, dx = \sec x$	**10.** $\int \csc x \cot x\, dx = -\csc x$
11. $\int \sec x\, dx = \ln|\sec x + \tan x|$	**12.** $\int \csc x\, dx = \ln|\csc x - \cot x|$
13. $\int \tan x\, dx = \ln|\sec x|$	**14.** $\int \cot x\, dx = \ln|\sin x|$
17. $\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	**18.** $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right),\quad a > 0$

- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.

Good Luck!

You may use this page for scratch-work or extra room.

All work on this page will be ignored unless you write and circle "see first scratch page" on the problem and you label your work.

1. (12 pts) Evaluate

(a) $\int \dfrac{\ln(x)}{x^3}\, dx$

(b) $\int \dfrac{\tan^2(x)\sec^2(x)}{\cos^2(x)}\, dx$

2. (12 pts) Evaluate

(a) $\int \dfrac{x^2 + 16}{x^3 + 2x^2} \, dx$

(b) $\int \dfrac{4x}{(x^2 + 2x + 5)^{3/2}} \, dx$

3. (12 pts) Evaluate

(a) $\displaystyle\int_1^5 \frac{\sqrt{x-1}}{x+3}\,dx$

(b) $\displaystyle\int \sqrt{9-x^2}\,dx$

4. (12 pts) The two parts below are unrelated.

(a) Evaluate the improper integral $\int_1^\infty \frac{1 + \sqrt{x}}{x^2} \, dx$.

(b) Consider the region under the function $y = f(x)$ from $x = 0$ to $x = 8$ which is shown below:

Correctly use the values from the graph to answer the following:
Use Simpson's rule with $n = 4$ subdivisions to estimate the volume of the solid obtained by rotating this region about the x-axis. In other words, estimate

$$\int_0^8 \pi (f(x))^2 \, dx.$$

Note: Your final answer will be a number times π, you can leave it in that form.
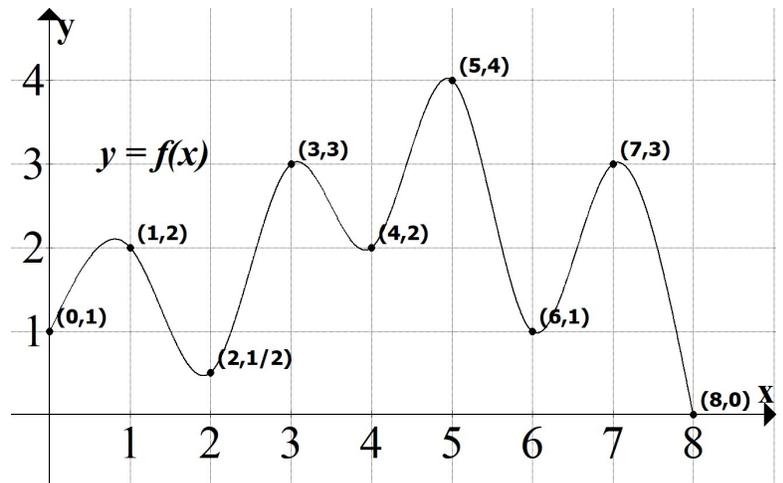
5. (12 pts) A leaky 2 lbs bucket is lifted from the ground to a height of 10 ft at a constant speed with a rope that weighs 0.3 lbs/ft. Initially the bucket contains 20 pounds of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 10-ft level.

   (a) How much work is done in lifting the leaky bucket alone (meaning not including the work to lift the rope)?

   (b) How much work is done in lifting the rope (not including the work to lift the bucket)?

   (c) How much work is done all together to lift the bucket and rope?

Total Work = _____ foot-pounds

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