Math 125
Exam 1
October 19, 2023

Name _____

Student ID #_____ Section _____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- This exam consists of a cover, a scratch sheet, five pages of questions, and another scratch sheet. If you put work on either scratch sheet and you want it to be graded, then you must clearly tell us in the problem to "see first scratch page" or "see last scratch page".
- You will have 80 minutes.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**) and one 8.5 by 11 inch sheet of handwritten notes (front and back). All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam. You may not listen to headphones or earbuds during the exam.
- **You must show your work.** The correct answer with no supporting work may result in no credit.
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\frac{7}{2} - \frac{3}{5} = \frac{29}{10}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.
- Also an answer containing an inverse trig inside of a trig function (such as $\cos(\sin^{-1}(x))$ or $\sin(2\cos^{-1}(x))))$ is not acceptable, instead show you can simplify using the triangle method from class.
- Unless otherwise indicated, when rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- There may be multiple versions, you have signed an honor statement, and cheating is a hassle for everyone involved. If we find that you give an answer that is only appropriate for the other version of the exam and there is no work to support your answer, then you will get a zero on the entire exam and your work will be submitted to the academic misconduct board. JUST DO NOT CHEAT.

GOOD LUCK!

You may use this page for scratch-work or extra room.
All work on this page will be ignored unless you write and circle "see first scratch page" on the problem and you label your work.

1. (12 pts) Evaluate the integrals. If you do a substitution in a definite integral problem (anywhere on this test), you must show me that you can appropriately change the bounds to get full credit. **Simplify your final answers**.

   (a) (4 pts) $\int \sqrt{x}\left(6+\frac{1}{x}\right) - \csc^2(5x)dx$

   (b) (4 pts) $\int_0^{\sqrt{\pi/6}} x\left(2\sin(x^2)+1\right)^4 \cos(x^2)\,dx.$

   (c) (4 pts) $\int_0^1 \frac{x^5}{2-x^3}\,dx$

2. (14 pts) Leave your answers in exact form, but **simplify your final answers**.

(a) (5 pts) Consider $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1+\frac{3i}{n}} \cdot \frac{3}{n}$. Rewrite this as an integral and evaluate the integral.

(b) Consider $F(x) = \int_{2x+1}^{3x^3} \frac{12}{1+t}\, dt$.

i. (5 pts) Find the equation for the tangent line to $F(x)$ at $x = 1$.

ii. (4 pts) Use the right-endpoint rule with $n = 2$ subdivisions to approximate the value of $F(3) = \int_{7}^{81} \frac{12}{1+t}\, dt$. (You do NOT have to simplify at all. Leave your answer expanded, it will be the sum of two products of numbers, just put the correct numbers in the correct places and stop.)

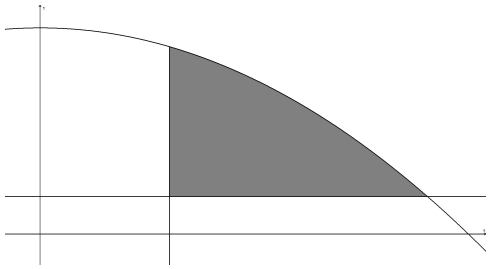
3. (14 pts) (The two problems below are NOT related). **Simplify your final answers**.

(a) (7 pts) $\int_1^3 \left|t^3 - 4t\right| \, dt$

(b) (7 pts) Find the area of the region bounded by $x = y^2$ and $x - 3 = 2y$. (You must sketch a picture of the region for full credit)

4. (12 pts) Consider the region R that is bounded on the left by $x = 1$, bounded on the bottom by $y = 2$ and bounded on the top by $y = 11 - x^2$ (shown below).

   (a) (3 pts) Find the area of the region.



   (b) (6 pts) Set up (but DO NOT EVALUATE) integrals for the VOLUME of the solid obtained by rotating R about the y-axis. Set it up using BOTH methods. Carefully include correct bounds and integrands (expect at least -2 per error, even small errors, so write your answers carefully!)

   VOLUME (using WASHERS) =

   VOLUME (using SHELLS) =

   (c) (3 pts) Using shells, set up (but DO NOT EVALUATE) an integral for the VOLUME of the solid obtained by rotating R about the vertical line $x = -3$.

   VOLUME (using SHELLS) =

5. (8 pts) A student is standing on the balcony of a building 320 feet above their math instructor. Assume everything thrown from the balcony has a constant acceleration of $a(t) = -32$ ft/s^2. Correctly integrate and find constants of integration to answer the following questions.

   (a) (4 pts) The student throws a tomato downward and it lands on the instructor's head in exactly 2.5 seconds. What was the initial velocity of the tomato? (include units)

   (b) (4 pts) A few moments later, the student throws a small water balloon straight downward (but with a much smaller initial downward velocity). At the moment the balloon lands on the instructor's head it is traveling at a velocity of -144 ft/sec. How many seconds did it take from the moment the balloon was thrown to when it lands on the instructor's head?

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