1. (12 pts) Evaluate the integrals. If you do a substitution in a definite integral problem (anywhere on this test), you must show me that you can appropriately change the bounds to get full credit. **Simplify your final answers.**

(a) (4 pts) $\int \underbrace{\sqrt{x}\left(6+\frac{1}{x}\right)}_{\text{SIMPLIFY}} - \csc^2(5x)dx = \int 6\sqrt{x} + x^{-1/2}\,dx - \int \csc^2(5x)dx$

$u = 5x$
$du = 5dx$

$= 6\cdot\frac{2}{3}x^{3/2} + 2x^{1/2} - \int \frac{1}{5}\csc^2(u)du$

$= 4x^{3/2} + 2\sqrt{x} + \frac{1}{5}\cot(u) + C$

$= \boxed{4x^{3/2} + 2\sqrt{x} + \frac{1}{5}\cot(5x) + C}$

(b) (4 pts) $\int_0^{\sqrt{\pi/6}} x\left(2\sin(x^2)+1\right)^4 \cos(x^2)\,dx.$

$u = 2\sin(x^2) + 1$
$du = 4x\cos(x^2)$
$\frac{1}{4}\,du = x\cos(x^2)dx$

$= \int_1^2 u^4 \frac{1}{4}\,du$

$= \frac{1}{20}u^5 \Big|_1^2$

$= \frac{1}{20}(2^5 - 1^5) = \boxed{\frac{31}{20} = 1.55}$

(c) (4 pts) $\int_0^1 \frac{x^5}{2-x^3}\,dx$

$u = 2 - x^3 \rightarrow x^3 = 2 - u$
$du = -3x^2\,dx$
$\frac{1}{-3x^2}\,du = dx$

$= \int_2^1 \frac{x^{\cancel{5}\,3}}{u}\frac{1}{-3\cancel{x^2}}\,du$

$= \frac{1}{3}\int_1^2 \frac{2-u}{u}\,du$

$= \frac{1}{3}\int_1^2 \frac{2}{u} - 1\,du$

$= \frac{1}{3}\left[2\ln|u| - u\Big|_1^2\right] = \frac{1}{3}\left[(2\ln(2) - 2) - (2\underbrace{\ln(1)}_{=0} - 1)\right]$

$= \boxed{\frac{1}{3}\left(2\ln(2) - 1\right)}$

2. (14 pts) Leave your answers in exact form, but **simplify your final answers**.

(a) (5 pts) Consider $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1+\frac{3i}{n}} \cdot \frac{3}{n}$. Rewrite this as an integral and evaluate the integral.

$$\Delta x = \frac{3}{n} \Rightarrow b-a=3$$

$$x_i = a + \frac{3i}{n} = 1 + \frac{3i}{n} \Rightarrow a=1 \Rightarrow b=4$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x = \int_1^4 \sqrt{x}\,dx = \frac{2}{3} x^{3/2} \Big|_1^4 = \frac{2}{3}\left[4^{3/2} - 1^{3/2}\right]$$

$$= \frac{2}{3}[8-1] = \boxed{\frac{14}{3}}$$

(b) Consider $F(x) = \int_{2x+1}^{3x^3} \frac{12}{1+t}\,dt$.

i. (5 pts) Find the equation for the tangent line to $F(x)$ at $x = 1$.

$$F(1) = \int_3^3 \frac{12}{1+t}\,dt = 0$$

$$F'(x) = \frac{12}{1+3x^3} \cdot 9x^2 - \frac{12}{1+2x+1} \cdot 2$$

$$F'(1) = \frac{12}{4} \cdot 9 - \frac{12}{4} \cdot 2 = 27 - 6 = 21$$

$$\boxed{y = 21(x-1)}$$

ii. (4 pts) Use the right-endpoint rule with $n = 2$ subdivisions to approximate the value of $F(3) = \int_7^{81} \frac{12}{1+t}\,dt$. (You can leave your answer expanded out without simplifying, it will be the sum of two products of numbers).

$$\Delta x = \frac{81-7}{2} = 37,$$

$$x_0 = 7,\ x_1 = 44,\ x_2 = 81$$

$$\boxed{\frac{12}{1+44} \cdot 37 + \frac{12}{1+81} \cdot 37}$$

$$= \frac{12 \cdot 37}{45} + \frac{12 \cdot 37}{82} \approx 15.281$$

3. (14 pts) (The two problems below are NOT related). **Simplify your final answers.**

(a) (7 pts) $\int_1^3 \left|t^3 - 4t\right| \, dt$

$t^3 - 4t = 0 \Rightarrow t(t^2-4) = 0 \Rightarrow t = 0, \pm 2$ ONLY $t=2$ IS IN INTERVAL

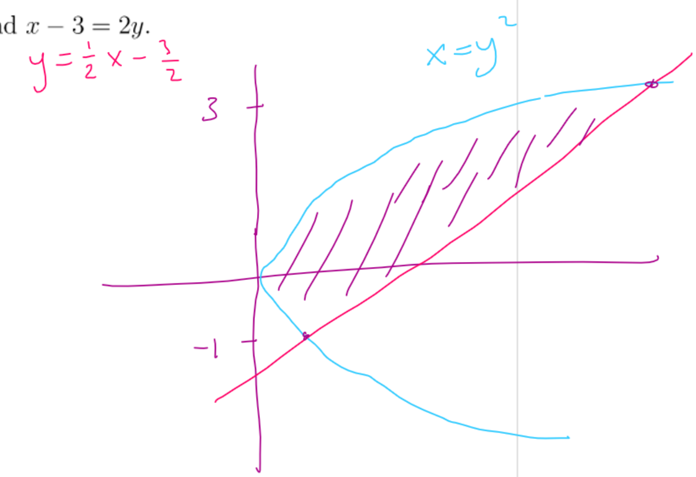
$$\int_1^2 t^3 - 4t \, dt = \tfrac{1}{4}t^4 - 2t^2 \Big|_1^2 = (4-8) - (\tfrac{1}{4} - 2) =$$
$$= -4 - -\tfrac{7}{4} = -9/4$$

$$\int_2^3 t^3 - 4t \, dt = \tfrac{1}{4}t^4 - 2t^2 \Big|_2^3 = \left(\tfrac{81}{4} - 18\right) - (4 - 8)$$
$$= \tfrac{81}{4} - 14 = \tfrac{81-56}{4} = \tfrac{25}{4}$$

$$\int_1^3 |t^3 - 4t| \, dt = \frac{9}{4} + \frac{25}{4} = \frac{34}{4} = \boxed{\frac{17}{2} = 8.5}$$

(b) (7 pts) Find the area of the region bounded by $x = y^2$ and $x - 3 = 2y$.

INTERSECT: $y^2 - 3 = 2y$
$\Rightarrow y^2 - 2y - 3 = 0$
$(y-3)(y+1) = 0$
$y = -1, y = 3$

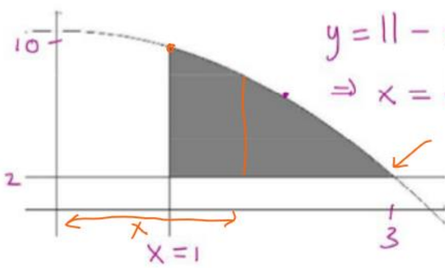


$$\int_{-1}^3 2y + 3 - y^2 \, dy$$
$$= y^2 + 3y - \tfrac{1}{3}y^3 \Big|_{-1}^3$$
$$= (9 + \cancel{9} - \cancel{9}) - (1 - 3 + \tfrac{1}{3})$$
$$= 9 - (-2 + \tfrac{1}{3}) = 11 - \tfrac{1}{3} = \boxed{\frac{32}{3}}$$

4. (12 pts) Consider the region R that is in the first quadrant and bounded by $y = 11 - x^2$, $x = 1$, and $y = 2$ (shown below).

(a) (3 pts) Find the area of the region.

$y = 2$ & $y = 11 - x^2$
INTERSECT WHEN $2 = 11 - x^2$
$\Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

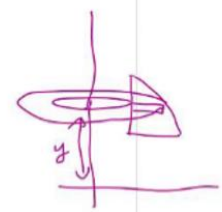


$y = 11 - x^2$
$\Rightarrow x = \sqrt{11 - y}$

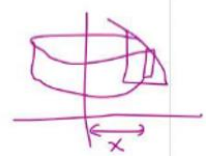
$$\int_1^3 11 - x^2 - 2 \, dx$$

$$\int_1^3 9 - x^2 \, dx = 9x - \frac{1}{3}x^3 \Big|_1^3$$
$$= (27 - 9) - (9 - \tfrac{1}{3})$$
$$= 9 + \frac{1}{3}$$
$$= \frac{28}{3}$$

(b) (6 pts) Set up (but DO NOT EVALUATE) integrals for the VOLUME of the solid obtained by rotating R about the y-axis. Set it up using BOTH methods. Carefully include correct bounds and integrands (expect at least -2 per error, even small errors, so write your answers carefully!)



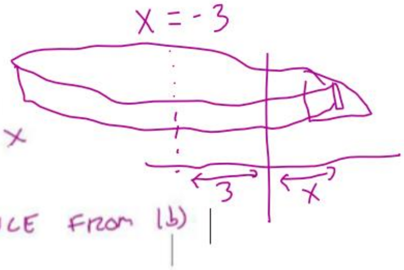
VOLUME (using WASHERS) = $\int_2^{10} \pi\left(\sqrt{11-y}\right)^2 - \pi(1)^2 \, dy$



VOLUME (using SHELLS) = $\int_1^3 2\pi\, x \underbrace{(11 - x^2 - 2)}_{9 - x^2} \, dx$

(c) (3 pts) Using shells, set up (but DO NOT EVALUATE) an integral for the VOLUME of the solid obtained by rotating R about the vertical line $x = -3$.

X = -3



VOLUME (using SHELLS) = $\int_1^3 2\pi(x + 3)(9 - x^2) \, dx$

↑
ONLY DIFFERENCE FROM 1(b)

5. (8 pts) A student standing balcony of a building 320 feet above their math instructor. Assume acceleration due to gravity is a constant $a(t) = -32$ ft/s^2. Correctly integrate and find constants of integration to answer the following questions.

(a) (4 pts) The student throws a tomato downward and it lands on the instructor's head in exactly 2.5 seconds. What was the initial velocity of the tomato? (include units)

$a(t) = -32$
$v(t) = -32t + C$
$h(t) = -16t^2 + Ct + D$

$h(0) = 320 \Rightarrow D = 320$
$h(2.5) = 0 \Rightarrow \underbrace{-16(2.5)^2}_{-100} + C(2.5) + 320 = 0$

$\Rightarrow 2.5C = -220$

$C = \frac{-220}{2.5} = \boxed{-88 \text{ ft/sec}}$

(b) (4 pts) A few moments later, the student throws a small water balloon straight downward (but with a different initial downward velocity). At the moment the balloon lands on the instructor's head it is traveling at a velocity of -144 ft/sec. How many seconds did it take from the moment the balloon was thrown to when it lands on the instructor's head?

$a(t) = -32$
$v(t) = -32t + C$
$h(t) = -16t^2 + Ct + 320$

$t = a$ = time to get to instructor's head

$\left.\begin{matrix} h(a) = 0 \\ v(a) = -144 \end{matrix}\right\} \Rightarrow \begin{matrix} -16a^2 + Ca + 320 = 0 \\ -32a + C = -144 \Rightarrow C = 32a - 144 \end{matrix}$

$\Rightarrow -16a^2 + (32a - 144)a + 320 = 0$ → or QUAD. FORMULA

$(16a^2 - 144a + 320 = 0)\frac{1}{16}$

$a^2 - 9a + 20 = 0$
$(a-4)(a-5) = 0$

$\left\{\begin{matrix} a = 4 \Rightarrow C = -16 \text{ ft/s} \leftarrow \text{YES} \\ \text{OR} \\ a = 5 \Rightarrow C = 16 \text{ ft/s} \; X \end{matrix}\right.$

↑ NOT DOWNWARD

SO $\boxed{a = 4 \text{ seconds}}$