

WORK EXAMPLES

SPRINGS

Spr 07) $x = \text{dist. beyond natural length}$

$$f(x) = kx = \text{force}$$

$$\text{Work} = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_a^b f(x) dx$$

FORCE DIST

$$\text{GIVEN: } \int_2^4 kx dx = 18 \Rightarrow \frac{k}{2} x^2 \Big|_2^4 = 18 \Rightarrow \frac{k}{2} (4^2 - 2^2) = 18$$

$$\Rightarrow 6k = 18 \Rightarrow k = 3$$

$$\text{FORCE} = 24 \text{ lbs} \Rightarrow 3x = 24 \Rightarrow x = 8 \text{ feet} \leftarrow \text{MAX DISTANCE.}$$

Spr 08) (a) $x = \text{dist. beyond natural length}$

$$\text{FORCE} = (1 \text{ kg}) \cdot (9.8 \text{ m/s}^2) = 9.8 \text{ N}$$

$$\text{WHEN } x = 0.03$$

$$\Rightarrow k(0.03) = 9.8 \Rightarrow k = 326.6$$

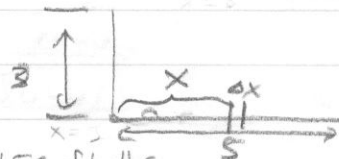
$$(b) \text{ work} = \int_{0.03}^{0.05} 326.6 x dx = \frac{326.6}{2} x^2 \Big|_{0.03}^{0.05} = \frac{326.6}{2} (0.05^2 - 0.03^2) = 0.2613 \text{ Joules}$$

LIFTING CHAINS/CABLES

Fall 06) Density = $\frac{150 \text{ lbs}}{10 \text{ ft}} = 15 \text{ lb/ft}$

The initial 3 ft of cable all moves up

$$10 \text{ feet. So work} = \underbrace{3 \text{ ft}}_{\text{FORCE}} \cdot \underbrace{15 \frac{\text{lb}}{\text{ft}} \cdot 10 \text{ ft}}_{\text{DIST}} = 450 \text{ ft-lbs}$$

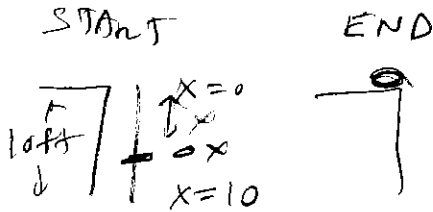


If a piece of cable is x units along the ground to start then the distance traveled will be $10 - x$.

$$\text{So work} = \lim_{n \rightarrow \infty} \sum_{i=1}^n 15x_i (10 - x_i) = \int_0^5 15(10 - x) dx = 150x - \frac{15}{2} x^2 \Big|_0^5 = 562.5 \text{ ft-lbs}$$

$$\text{TOTAL} = 450 + 562.5 = 1012.5 \text{ ft-lbs}$$

Spring 10 DENSITY = $\frac{4 \text{ lbs}}{10 \text{ ft}} = 0.4 \frac{\text{lbs}}{\text{ft}}$



FORCE = $0.4 \Delta x$
DIST = x

$\int_0^{10} x \cdot 0.4 dx$

$0.2 x^2 \Big|_0^{10} = 0.2 (10)^2 = \boxed{20 \text{ ft}\cdot\text{lb}}$

$20 \text{ ft}\cdot\text{lb} < 40 \text{ ft}\cdot\text{lb}$

TRUE!

FALL 2010

START $y = 35 \text{ m}$



(a) **BUCKET**

| y | MASS OF WATER | Force | $y=0 \text{ m}$ | y |
|-----|---------------|----------------|--|---|
| 0 | 20 | $9.8 \cdot 20$ | $0.1 \text{ kg} \leftrightarrow 1 \text{ sec}$ | $0.5 \text{ m} \leftrightarrow 1 \text{ sec}$ |

WATER MASS = $20 - 0.2 y$

$\frac{0.1 \text{ kg}}{1 \text{ sec}} \cdot \frac{1 \text{ sec}}{0.5 \text{ m}} = \frac{0.1 \text{ kg}}{0.5 \text{ m}}$

(b) TWO PARTS

Bucket Force = $9.8(2 + 20 - 0.2y)$ N

DIST = $0.5y$

$\boxed{0.2 \frac{\text{kg}}{\text{m}}}$

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$$\boxed{\text{WATER DRUCK}} = \int_0^{35} 9.8(22 - 0.2y) dy$$

$$= 9.8(22y - 0.1y^2 \Big|_0^{35})$$

$$= 6345.5 \text{ Joules}$$

CABLE

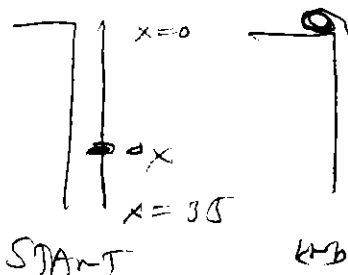
$$\text{FORCE} = 9.8 \cdot 0.2 \cdot x$$

$$\text{DIST} = x$$

$$\int_0^{35} x \cdot 9.8 \cdot 0.2 dx$$

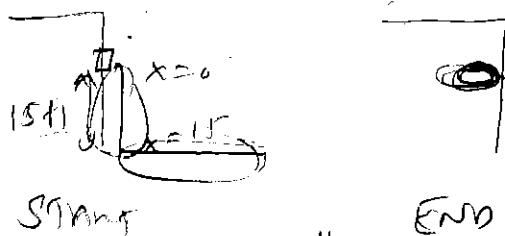
$$= 9.8 \cdot 0.1 x^2 \Big|_0^{35}$$

$$= 0.98 (35)^2 = 1200.5 \text{ Joules}$$



$$\boxed{\text{TOTAL} = 7546 \text{ Joules}}$$

Spring 2012



$$\int_0^{15} x \cdot 2 dx$$

$$= x^2 \Big|_0^{15}$$

$$+ 70 \cdot 15$$

$$+ 1050 = 225 + 1050 = \boxed{1275}$$

35 ft. $\cdot \frac{2 \text{ lb}}{\text{ft}} = 70 \text{ lbs}$
 lifted 15 ft

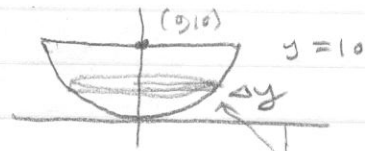
PUMPS

Fall 05 | $x^2 + (y-10)^2 = 10^2$
 $\Rightarrow x = \sqrt{100 - (y-10)^2}$

DIST TO TOP = $10 - y$

FORCE = DENSITY · VOLUME OF CROSS-SECTIONAL SLICE

= $1000 \cdot 9.8 \cdot \pi (\sqrt{100 - (y-10)^2})^2 \cdot \Delta y$
DENSITY N/m^3 VOLUME

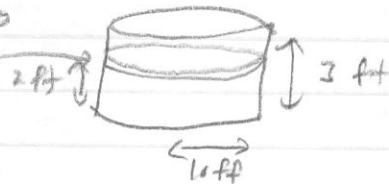


Work = $\int_0^{10} 9800\pi (100 - (y-10)^2) (10-y) dy$

Spr. 06 | DIST TO TOP = x TOP $x=0$

FORCE = $62.5 \cdot \pi (10)^2 \cdot \Delta x$
DENSITY VOLUME

BOTTOM $x=7$

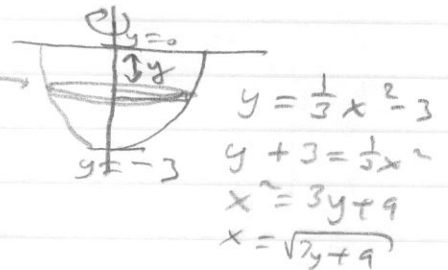


Work = $\int_7^3 62.5\pi (10)^2 x dx$

= $6250\pi \left. \frac{1}{2}x^2 \right|_7^3 = \frac{6250\pi}{2} (9-49) = \boxed{25000\pi} \text{ ft-lbs}$

Win 07 | DIST TO TOP = y

FORCE = $3.7 \cdot 1000 \cdot \pi (\sqrt{3y+9})^2 \Delta y$
 N/m^3 Volume

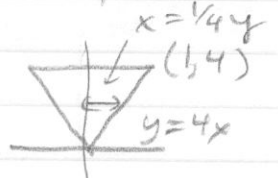
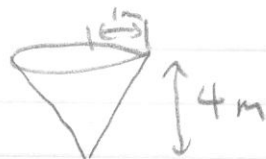


Work = $\int_{-3}^0 3700\pi (3y+9)(y) dy$

= $3700\pi \int_{-3}^0 (3y^2 + 9y) dy$
 = $3700\pi \left(y^3 + \frac{9}{2}y^2 \right) \Big|_{-3}^0$
 = $3700\pi \left(0 - \left((-3)^3 + \frac{9}{2}(-3)^2 \right) \right) = 3700\pi \left(+27 - \frac{27}{2} \right)$
 = $3700\pi \cdot \frac{27}{2} = \boxed{49950\pi \approx 156922.55 \text{ J}}$

Fall 07 | DIST TO TOP = $4 - y$

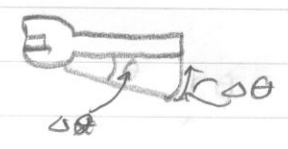
FORCE = $9.8 \cdot 1000 \cdot \pi \left(\frac{1}{4}y\right)^2 \Delta y$
DENSITY Volume



Work = $\int_0^3 9800\pi \frac{1}{16}y^2 (4-y) dy = \dots = \boxed{\frac{77175\pi}{3} \approx 30306.6 \text{ J}}$
 I'll leave this to you

OTHER

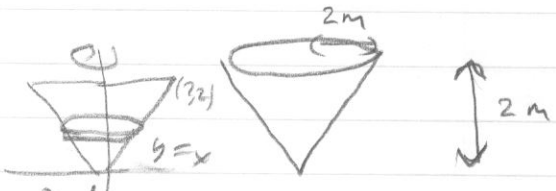
Fall 05 | Force = $3 + \tan^2(\theta)$ Newtons
 DIST = $r \cdot d\theta = \underbrace{0.3}_{\text{meters}} d\theta$



$$\begin{aligned} \text{Work} &= \int_0^{\pi/4} (3 + \tan^2(\theta)) 0.3 d\theta \\ &= \int_0^{\pi/4} 0.9 + 0.3 \tan^2(\theta) d\theta \\ &= \int_0^{\pi/4} 0.9 + 0.3 \sec^2 \theta - 0.3 d\theta \\ &= 0.6\theta + 0.3 \tan \theta \Big|_0^{\pi/4} \\ &= (0.6\pi/4 + 0.3) - 0 = \boxed{0.15\pi + 0.3 \approx 0.77124 \text{ J}} \end{aligned}$$

$\tan^2 \theta = \sec^2 \theta - 1$

Win 09 | DIST TO TOP = $2 - y$
 FORCE = $\underbrace{9.8 \cdot 1676}_{\text{DENSITY}} \cdot \underbrace{\pi (y)^2}_{\text{radius}} dy$

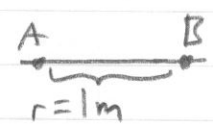


(a) $\text{Work} = \int_1^2 16424.8 \pi y^2 (2-y) dy$
 $= 16424.8 \pi \int_1^2 (2y^2 - y^3) dy = \dots = \boxed{47300 \text{ J}}$

learn to you

(b) $\text{Work} = \int_0^1 16424.8 \pi y^2 (2-y) dy = \dots = \boxed{21500 \text{ J}}$

Fall 09 | Force = $-\frac{9 \times 10^9}{r^2}$



(a) $\text{Work} = \int_1^2 \frac{9 \times 10^9}{r^2} dr$
 $= 9 \times 10^9 \left(-\frac{1}{r}\right) \Big|_1^2 = 9 \times 10^9 \left(-\frac{1}{2} + 1\right) = \frac{9 \times 10^9}{2}$
 $= \boxed{4.5 \times 10^9 \text{ J}}$

(b) $\text{Work} = \int_1^{\infty} \frac{9 \times 10^9}{r^2} dr = \lim_{t \rightarrow \infty} 9 \times 10^9 \int_1^t \frac{1}{r^2} dr$
 $= \lim_{t \rightarrow \infty} 9 \times 10^9 \left[-\frac{1}{r}\right]_1^t$
 $= \lim_{t \rightarrow \infty} 9 \times 10^9 \left[-\frac{1}{t} + 1\right] = \boxed{9 \times 10^9 \text{ J}}$

From LATER IN THE COURSE

Fall 111

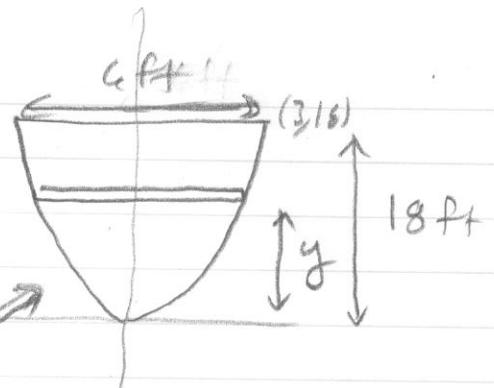
$$y = ax^2 + bx + c$$

$b=0$ because symmetric

$c=0$ because it goes through the origin

$$y = ax^2$$

$$18 = a(3)^2 \Rightarrow a = 2$$



$$y = 2x^2$$

$$x = \sqrt{y/2}$$

$$\text{DIST moved} = 15 + y$$

$$\text{FORCE} = 3 \cdot 2\sqrt{y/2} \cdot dy$$

| y | DIST moved |
|---|------------|
| 0 | 15 ft |
| 1 | 16 ft |
| 2 | 17 ft |
| y | 15+y ft |

$$\int_0^{18} 3 \cdot 2\sqrt{y/2} (15+y) dy$$

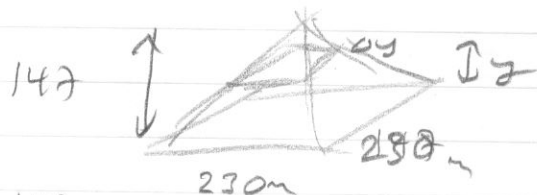
$$= \frac{6}{\sqrt{2}} \int_0^{18} 15y^{1/2} + y^{3/2} dy = \dots = \frac{2784}{5} = 5572.8 \text{ ft}\cdot\text{lbs}$$

Fall 112 $s(y) = 2 \left(\frac{-115}{147} y + 115 \right)$

DIST TO LIFT = y

FORCE = $9.8 \cdot 2360 \cdot (s(y))^2 \cdot dy$

DENSITY

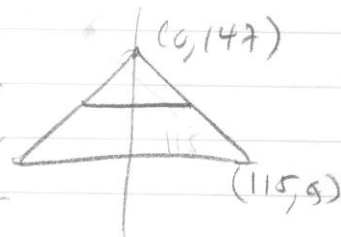


$$\frac{147-0}{0-115}$$

$$y = \frac{-147}{115} x + 147$$

$$\frac{-115}{147} (y - 147) = x$$

$$x = -\frac{115}{147} y + 115$$



$$\int_0^{147} 9.8 \cdot 2360 \cdot \left(2 \left(\frac{-115}{147} y + 115 \right) \right)^2 \cdot y dy = \dots = 2.20317 \times 10^{12} \text{ J}$$