Math 125
Exam 2
November 14, 2024

Name _____

Student ID #_____   Section _____

- This exam consists of a cover, a scratch sheet, five pages of questions, and another scratch sheet. If you put work on either scratch sheet and you want it to be graded, then you must clearly tell us in the problem to "see first scratch page" or "see last scratch page".

- Turn off and stow away all cell phones, smart watches, music players, and other similar devices.

- You may use one $8.5'' \times 11''$ sheet of handwritten notes. You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.

- You must **show your work**. You will NOT get credit if there is no or incomplete work, even if your final answer is correct.

- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.

- **Special Note for Trig Sub**: An answer containing an inverse trig inside of a trig function, such as $\cos(\sin^{-1}(x))$ or $\sin(2\cos^{-1}(x))$, is NOT acceptable, instead show that you can simplify using the triangle method from class.

- You may use directly the integral formulas in the table below. You must **show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas   Constants of integration have been omitted.

1. $\int x^n \, dx = \dfrac{x^{n+1}}{n+1}$ $(n \neq -1)$ 2. $\int \dfrac{1}{x} \, dx = \ln|x|$

3. $\int e^x \, dx = e^x$ 4. $\int b^x \, dx = \dfrac{b^x}{\ln b}$

5. $\int \sin x \, dx = -\cos x$ 6. $\int \cos x \, dx = \sin x$

7. $\int \sec^2 x \, dx = \tan x$ 8. $\int \csc^2 x \, dx = -\cot x$

9. $\int \sec x \tan x \, dx = \sec x$ 10. $\int \csc x \cot x \, dx = -\csc x$

11. $\int \sec x \, dx = \ln|\sec x + \tan x|$ 12. $\int \csc x \, dx = \ln|\csc x - \cot x|$

13. $\int \tan x \, dx = \ln|\sec x|$ 14. $\int \cot x \, dx = \ln|\sin x|$

17. $\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$ 18. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right)$, $a > 0$

- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.

Good Luck!

You may use this page for scratch-work or extra room.

All work on this page will be ignored unless you write and circle "see first scratch page" on the problem and you label your work.

1. (12 pts) Evaluate

(a) $\int \dfrac{5x^2 + x + 8}{x^3 + 4x}\, dx$

Answer = _____

(b) $\int \dfrac{1}{\sqrt{4x^2 + 16x + 52}}\, dx$

Answer = _____

2. (12 pts) Evaluate

(a) $\int x^2 \tan^{-1}(x)\, dx$

Answer = _____

(b) $\int \dfrac{x+4}{x(1+\sqrt{x})^2}\, dx$

Answer = _____

3. (12 pts)

(a) Evaluate $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx$

Answer = _____

(b) Find the average value of $f(x) = x^2 e^{x/2}$ from $x = 0$ to $x = 4$.

Average Value = _____

4. (12 pts) Consider the infinite region bounded by $f(x) = \dfrac{1}{x^2 + 1}$, $y = 0$, $x = \sqrt{3}$, shown below.

For both parts, if the integral diverges, then write "diverges" and show work. If it converges, show work and give the value simplified as much as possible.

(a) Evaluate the following integral to find the area: $\displaystyle\int_{\sqrt{3}}^{\infty} \dfrac{1}{x^2 + 1}\, dx$.

f(x)

$\sqrt{3}$
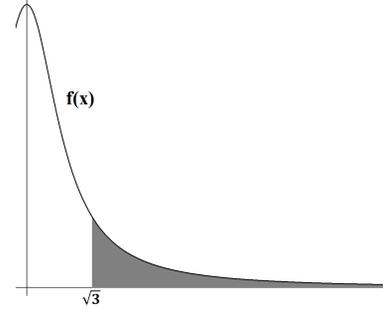
Answer (or say diverges) = _____

(b) Set up an improper integral that represents the volume of the solid obtained by rotating this region around the y-axis (using shells!). Then evaluate the integral.

Integral Representing the Volume = _____

Evaluation of integral (or say diverges) = _____

5. (12 pts)

(a) Given that $\int_1^9 g(x)\,dx = 6$, find $\int_1^3 xg(x^2) + 3x^2\,dx$

Answer = _____

(b) A trough-shaped tank of length 6 feet has vertical cross sections given by $y = x^2$ (as shown). The top of the tank is at $x = \pm\sqrt{12}$ ft. The tank currently has water up to a height of 4 feet.

Find the work required to pump all the water currently in the tank to the top of the tank. (Use 62.5 lbs/ft^3 for the density of water.)

$y = x^2$

4

$x = \sqrt{12}$

$y = x^2$

4

6
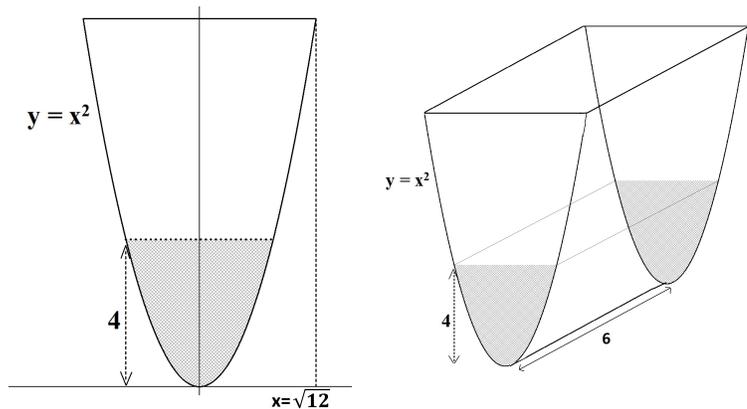
Work (include units) = _____

You may use this page for scratch-work or extra room.

All work on this page will be ignored unless you write and circle "see last scratch page" on the problem and you label your work.