1. ($\cancel{12}$ 13 pts) Parts (a), (b), and (c) are not related.

(a) (4 pts) Evaluate $\int \frac{(x+3)(x-3)}{\sqrt{x}} + \sec(2x)\tan(2x)\,dx$

SEPARATE & SIMPLIFY

$$= \int \frac{x^2-9}{\sqrt{x}}\,dx + \int \sec(2x)\tan(2x)\,dx$$

$u = 2x$
$du = 2dx$

$$= \int x^{3/2} - 9x^{-1/2}\,dx + \frac{1}{2}\int \sec(u)\tan(u)\,du$$

$$= \frac{2}{5}x^{5/2} - 18x^{1/2} + \frac{1}{2}\sec(u) + C$$

V2: 3 Answer = $\boxed{\frac{2}{5}x^{5/2} - 18\sqrt{x} + \frac{1}{2}\sec(2x) + C}$

(b) ($\cancel{4}$ 5 pts) Evaluate $\int \frac{e^x(e^x+2)}{e^x-1}\,dx$

$u = e^x - 1 \rightarrow e^x = u+1$
$du = e^x\,dx$
$\frac{1}{e^x}du = dx$

$$\int \frac{e^x(e^x+2)}{u}\frac{1}{e^x}\,du$$

4 IN OTHER VERSION

$$\int \frac{u+1+2}{u}\,du = \int 1 + \frac{3}{u}\,du = u + 3\ln|u| + C$$

$$= \boxed{e^x - 1 + 3\ln|e^x-1| + C}$$
$$= \boxed{e^x + 3\ln|e^x-1| + D}$$

BOTH ACCEPTABLE

Answer = __________

V2: 7

(c) (4 pts) Express the following Riemann sum as a definite integral: $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{5+\sqrt{1+\frac{3i}{n}}}\cdot\frac{3}{n}$.

Do NOT evaluate, just give the integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{5+\sqrt{x_i}}\Delta x$$

$\Delta x = \frac{b-a}{n} = \frac{3}{n} \Rightarrow b-a=3$

$x_i = a + \frac{3i}{n} \Rightarrow a = 1 \Rightarrow b = 4$

ANOTHER CORRECT ANSWER

$a = 0 \Rightarrow b = 3$

$$\Rightarrow \int_0^3 \frac{1}{5+\sqrt{1+x}}\,dx$$

Integral = $\boxed{\int_1^4 \frac{1}{5+\sqrt{x}}\,dx}$

7 IN OTHER VERSION

ANS $6-\frac{1}{\pi}$

2. ($\cancel{10}$ 9 pts) Parts (a) and (b) are not related. $\boxed{V2: 3}$

(a) ($\cancel{5}$ 4 pts) Let $g(x) = \int_{\frac{x}{\pi}}^{\tan(x)} 6\sqrt{t}\,dt$. Find the derivative of $g(x)$ at $x = \pi/4$.

Simplify the final answer as much as possible.

$$g'(x) = 6\sqrt{\tan(x)} \cdot \sec^2(x) - 6\sqrt{\frac{x}{\pi}} \cdot \frac{1}{\pi}$$

$$g'(\pi/4) = 6\sqrt{\tan(\pi/4)} \cdot \sec^2(\pi/4) - 6\sqrt{\frac{\pi/4}{\pi}}\ \frac{1}{\pi}$$

$$= 6 \cdot \sqrt{1} \cdot \frac{1}{(\sqrt{2}/2)^2} - 6 \cdot \frac{1}{2} \cdot \frac{1}{\pi}$$

$$= 6 \cdot \frac{1}{(1/2)} - \frac{3}{\pi}$$

$$= 12 - \frac{3}{\pi}$$

$6-\frac{1}{\pi}$ IN OTHER VERSION

$g'(\pi/4) =$ $12 - \frac{3}{\pi}$

(b) (5 pts) Use substitution to evaluate the following integral: $\int_2^3 x\sqrt[3]{x-2}\,dx$

Show your work in correctly changing the bounds.

$u = x-2 \rightarrow x = u+2$

$du = dx$

$$\int_0^1 (u+2)\,u^{1/3}\,du$$

$$\int_0^1 u^{4/3} + 2u^{1/3}\,du$$

$$\frac{3}{7}u^{7/3} + \frac{6}{4}u^{4/3}\Big|_0^1$$

$$= \left(\frac{3}{7} + \frac{3}{2}\right) - (0) =$$

$$= \frac{6}{14} + \frac{21}{14} = \frac{27}{14}$$

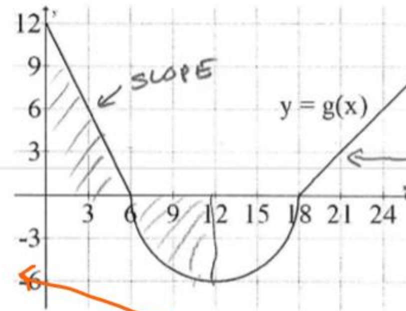
Answer = $\frac{3}{7} + \frac{3}{2} = \frac{27}{14}$

3. (14 pts) Part (a) and (b) below not related.

(a) (8 pts) The graph of $g(x)$ shown consists of two straight lines and a semicircle. Let $f(x) = \int_0^x g(t)\,dt$ and answer the following questions.

i. Use the graph to compute the values:

NET AREA



(1) $f(12) = \boxed{36 - 9\pi}$

12 6 $\frac{1}{2}(12)(6)$ 36 $-$ $\frac{1}{4}\pi(6)^2$ 9π

V2 SWAP

(2) $f'(21) = \boxed{3}$ $f'(3) = 6$

$g(21)$

(3) $f''(3) = \boxed{-2}$ $F''(21) = \frac{6}{6} = 1$ IN OTHER VERSION

$g'(3) = \text{slope}$ $\frac{\text{RISE}}{\text{RUN}} = \frac{-12}{6} = -2$

V2 MIN

ii. At what value of x does $f(x)$ have a local max?

min

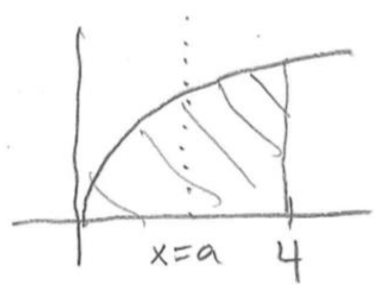
$x = \boxed{6}$ $x = 18$ IN OTHER VERSION

$f'(x)$ NEEDS TO LOOK LIKE THIS + −

(b) (6 pts) Consider the region, R, bounded by $y = \sqrt{x}$, $x = 4$ and the x-axis. Find the area of this region. Then find the value of a such that vertical line $x = a$ divides this region in half.

TOTAL AREA $= \int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{2}{3}(4)^{3/2} = \frac{16}{3}$

WANT $\int_0^a \sqrt{x}\,dx = \frac{1}{2}\cdot\frac{16}{3}$



$\frac{2}{3}x^{3/2}\Big|_0^a = 8/3$

$\Rightarrow \frac{2}{3}a^{3/2} = 8/3$

$\Rightarrow a^{3/2} = 4$

$\Rightarrow a = 4^{2/3} = (16)^{1/3}$

Total Area of the Entire Region = $16/3$

Value of a = $4^{2/3} = 16^{1/3} = 2\cdot 2^{1/3}$

ALL ACCEPTABLE

4. (12 pts) Parts (a) and (b) below are not related.

(a) (6 pts) Evaluate $\int_1^4 \left|\frac{2x^3-16}{x^2}\right| dx$

$$\frac{2x^3-16}{x^2} = 0 \Rightarrow 2x^3 = 16$$
$$\Rightarrow x^3 = 8$$
$$\Rightarrow x = 2$$

$$\int_1^2 \frac{2x^3-16}{x^2} dx = \int_1^2 2x - 16x^{-2} dx$$
$$= x^2 + 16x^{-1}\Big|_1^2 = (4+8) - (1+16)$$
$$= 12 - 17 = -5$$

$$\int_2^4 \frac{2x^3-16}{x^2} dx = x^2 + 16x^{-1}\Big|_2^4 = (16+4) - (4+8)$$
$$= 20 - 12 = 8$$

CONCLUSION

$$5 + 8 = 13$$

Answer = 13

(b) (6 pts) A tomato is thrown downward from the top of a 90 foot building **on the moon**. At $t = 2$ seconds, the tomato hits the ground. Assume the tomato accelerates at a constant 5 feet/sec^2 downward on the moon and assume there is no air resistance.

Find the function for the height, $h(t)$, of the tomato above the ground t seconds after being thrown and give the initial velocity of the tomato in feet/sec.

$a(t) = -5$, $v(t) = -5t + c$, $h(t) = -\frac{5}{2}t^2 + ct + D$

$h(0) = 90 \Rightarrow D = 90$

$h(2) = 0 \Rightarrow -\frac{5}{2}(2)^2 + c(2) + 90 = 0$

$$-10 + 2c + 90 = 0$$
$$2c = -80$$
$$c = -40$$

$h(t) = -\frac{5}{2}t^2 - 40t + 90$

Initial velocity $= v(0) =$ -40 feet/sec

5. (12 pts) **For all parts below**, consider the region bounded by $y = x^2$ and $y = 3 - 2x$, shown below.

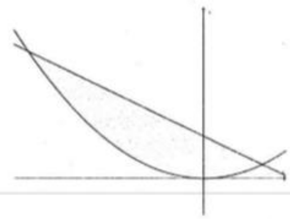
Please be clear about your bounds and integrands in each set-up.

(a) (1 pt) Find the two (x, y) points of intersection of these curves.

$x^2 = 3 - 2x \Rightarrow x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = 1,\ x = -3$



List both: $(x, y) =$ $(1,1)\quad(-3,9)$

(b) (3 pts) Set up an integral (using dx) that represent the **AREA** of this region. Include the correct bounds. (Do NOT evaluate).

Area (using dx) = $\displaystyle\int_{-3}^{1} 3 - 2x - x^2\, dx$ $\quad \frac{32}{3}$

(c) (4 pts) Set up integrals (using dy) that represent the **AREA** of this region. This will require you add two integrals. (Do NOT evaluate).

Area (using dy) first integral = $\displaystyle\int_0^1 \sqrt{y} - -\sqrt{y}\, dy = \int_0^1 2\sqrt{y}\, dy$

$= 4/3$

+

Area (using dy) second integral = $\displaystyle\int_1^9 \frac{y-3}{-2} - -\sqrt{y}\, dy = \int_1^9 \frac{3}{2} - \frac{y}{2} + \sqrt{y}\, dy$

$= 28/3$

V2: = 1

(d) (4 pts) Set up an integral for the **VOLUME** of the solid obtained by rotating R about horizontal line $y = -2$. (Do NOT evaluate).

Volume = $\displaystyle\int_{-3}^{1} \pi\,(3 - 2x + 2)^2 - \pi\,(x^2 + 2)^2\, dx$

$= \pi \displaystyle\int_{-3}^{1} (5 - 2x)^2 - (x^2 + 2)^2\, dx$ $\quad = \frac{576}{5}\pi$

4 $\quad$ 1

IN OTHER VERSION