Math 125
Exam 2
November 16, 2023

Name _____

Student ID #_____ Section _____

- This exam consists of a cover, a scratch sheet, five pages of questions, and another scratch sheet. If you put work on either scratch sheet and you want it to be graded, then you must clearly tell us in the problem to "see first scratch page" or "see last scratch page".
- Turn off and stow away all cell phones, smart watches, music players, and other similar devices.
- You may use one $8.5'' \times 11''$ sheet of handwritten notes. You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- You must **show your work**. You will NOT get credit if there is no or incomplete work, even if your final answer is correct.
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.
- ***Special Note for Trig Sub***: An answer containing an inverse trig inside of a trig function, such as $\cos(\sin^{-1}(x))$ or $\sin(2\cos^{-1}(x))$, is NOT acceptable, instead show that you can simplify using the triangle method from class.
- You may use directly the integral formulas in the table below. You must **show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x} dx = \ln|x|$
3. $\int e^x dx = e^x$
4. $\int b^x dx = \frac{b^x}{\ln b}$
5. $\int \sin x \, dx = -\cos x$
6. $\int \cos x \, dx = \sin x$
7. $\int \sec^2 x \, dx = \tan x$
8. $\int \csc^2 x \, dx = -\cot x$
9. $\int \sec x \tan x \, dx = \sec x$
10. $\int \csc x \cot x \, dx = -\csc x$
11. $\int \sec x \, dx = \ln|\sec x + \tan x|$
12. $\int \csc x \, dx = \ln|\csc x - \cot x|$
13. $\int \tan x \, dx = \ln|\sec x|$
14. $\int \cot x \, dx = \ln|\sin x|$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.

Good Luck!

You may use this page for scratch-work or extra room.
All work on this page will be ignored unless you write and circle "see first scratch page" on the problem and you label your work.

1. (12 pts) Evaluate

(a) $\int x \sec^4(x^2)\, dx$

(b) $\int \dfrac{x^2 - 2x + 12}{x^3 + 3x^2}\, dx$

2. (12 pts) Evaluate

(a) $\int \tan^{-1}(x)\,dx$

(b) $\int \frac{x^2}{(9-x^2)^{3/2}}\,dx$

3. (12 pts) Evaluate

(a) $\displaystyle\int \frac{1}{\sqrt{x^2+6x+5}}\,dx$

(b) $\displaystyle\int_1^8 \frac{x^{1/3}}{x+x^{1/3}}\,dx.$

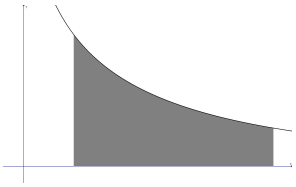
4. (12 pts) The two parts below are unrelated.

(a) Find the **average value** of the function $f(x) = (\ln(x))^2$ on the interval $x = 1$ to $x = e$. (Your final answer will involved the number e, leave it in simplified exact form).

Average value of $f(x) =$ _____

(b) The region shown is bounded by the curves $y = e^{-\sqrt{x}}$, $y = 0$, $x = 1$ and $x = 5$. Assume this region is rotated about the y-axis to form a solid.
Set up the integral for the volume of the solid (do not evaluate).

Then use Simpson's rule with $n = 4$ to approximate the value of the integral.
(Leave the Simpson's rule approximation expanded and unsimplified).

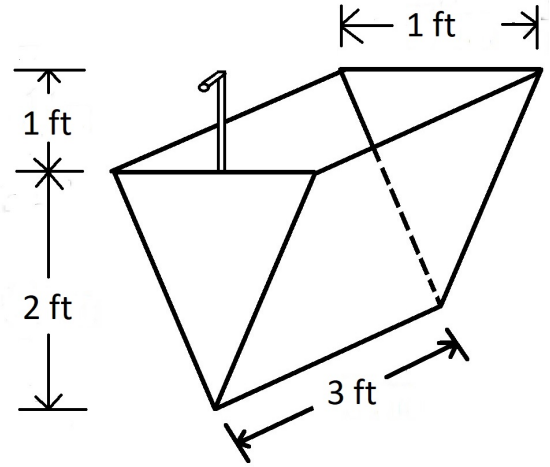


Volume Integral =

Approx =

5. (12 points) The trough-shaped tank shown is full of water and all the water is going to be pumped up and out of a spout. The dimensions are as indicated in the picture. Note the top of the spout is 1 foot above the top of the full tank.

   Find the work required to pump all the water out of the spout
   (Use 62.5 lbs/ft^3 for the density of water.)



Work (include units) = _____

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