1. (12 pts) Evaluate the integrals. If you do a substitution in a definite integral problem (anywhere on this test), you must show me that you can appropriately change the bounds to get full credit. **Simplify your final answers.**

(a) $\int (x^2+5)^2 - \sec(3x)\tan(3x)dx$

$$\int x^4 + 10x^2 + 25\, dx - \int \sec(3x)\tan(3x)dx$$

$$\boxed{\frac{1}{5}x^5 + \frac{10}{3}x^3 + 25x - \frac{1}{3}\sec(3x) + C}$$

(b) $\int_1^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}dx$

$u = \sqrt{x} + 1$

$du = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\, du = dx$

$x = 4 \rightarrow u = \sqrt{4} + 1 = 3$

$x = 1 \rightarrow u = \sqrt{1} + 1 = 2$

$$= \int_2^3 \frac{1}{\cancel{\sqrt{x}}\, u^2}\, 2\cancel{\sqrt{x}}\, du$$

$$= 2\left(-\frac{1}{u}\Big|_2^3\right)$$

$$= 2\left(-\frac{1}{3} - -\frac{1}{2}\right) = -\frac{2}{3} + 1 = \boxed{\frac{1}{3}}$$

(c) $\int \frac{3x^5}{2+x^3}dx$

$u = 2 + x^3$ $\quad x^3 = u - 2$

$du = 3x^2 dx$

$\frac{1}{3x^2}du = dx$

$$\int \frac{3x^{\cancel{5}}}{u} \frac{1}{3\cancel{x^2}}du$$

$$\int \frac{u-2}{u}du$$

$$\int 1 - \frac{2}{u}du = u - 2\ln|u| + C$$

$D = 2 + C$

$$\boxed{\begin{aligned} &= (2+x^3) - 2\ln|2+x^3| + C \\ &= x^3 - 2\ln|2+x^3| + D \end{aligned}}$$

2. (12 pts) The three parts below are not related.

(a) Express the following Reimann sum limit as a definite integral, then evaluate the integral:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{3n}\cos\left(\frac{\pi i}{3n}\right)$$

$\Delta x = \frac{b-a}{n} = \frac{\pi}{3n} \Rightarrow b-a = \pi/3$

$x_i = a + i\Delta x = 0 + \frac{\pi i}{3n}$

$a=0$

$b=\pi/3$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\cos(x_i)\Delta x$

$= \int_0^{\pi/3}\cos(x)\,dx$

$= \sin(x)\Big|_0^{\pi/3} = \sin(\pi/3) - \sin(0) = \frac{\sqrt{3}}{2} - 0$

Value of Integral = $\boxed{\sqrt{3}/2}$

(b) Find the value of the derivative of the function $F(x) = \int_{3\sqrt{x}}^{2x+1} t^2 e^t\,dt$ at $x = 1$.

$F'(x) = (2x+1)^2 e^{2x+1}\cdot 2 - 9x e^{3\sqrt{x}}\cdot\frac{3}{2}\frac{1}{\sqrt{x}}$

$\Rightarrow F'(1) = 9e^3\cdot 2 - 9e^3\cdot\frac{3}{2}$

$= 9e^3(2-3/2) = \frac{9e^3}{2}$

$F'(1) = \boxed{\frac{9e^3}{2}}$

(c) Use the midpoint rule with $n = 3$ subdivisions to approximate $\int_4^{10} e^{t^2}\,dt$. (You do NOT have to simplify. Leave your answer expanded.)

$\Delta x = \frac{10-4}{3} = 2$ $\quad x_0 = 4,\ x_1 = 6,\ x_2 = 8,\ x_3 = 10$

$\bar{x}_1 = 5,\ \bar{x}_2 = 7,\ \bar{x}_3 = 9$

$\int_4^{10} e^{t^2}\,dt \approx e^{(5)^2}\cdot 2 + e^{(7)^2}\cdot 2 + e^{(9)^2}\cdot 2$

$= (e^{25} + e^{49} + e^{81})\cdot 2$

3. (12 pts) (The two problems below are NOT related).

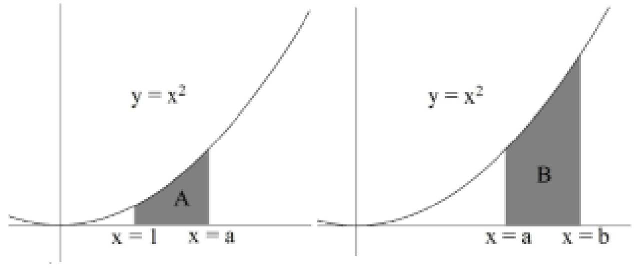
(a) For positive constants a and b, consider the area under $y = x^2$ and above the x-axis from $x = 1$ to $x = a$ and from $x = a$ to $x = b$, shown below. If the area of region B is two times the area of region A, give the formula for b in terms of a.

$A = \int_1^a x^2\,dx = \frac{1}{3}x^3 \Big|_1^a$

$= \frac{1}{3}a^3 - \frac{1}{3}$

$B = \int_a^b x^2\,dx = \frac{1}{3}x^3 \Big|_a^b$

$= \frac{1}{3}b^3 - \frac{1}{3}a^3$



$B = 2A \Rightarrow \frac{1}{3}b^3 - \frac{1}{3}a^3 = \frac{2}{3}a^3 - \frac{2}{3} \Rightarrow \frac{1}{3}b^3 = a^3 - 2/3$

$b^3 = 3a^3 - 2$

$b = \boxed{(3a^3 - 2)^{1/3}}$

(b) Find the area of the region bounded by $x = 2y^2$ (PARABOLA) and $x - 4 = 2y$ (LINE). (You must sketch a picture of the region for full credit)

$x = 2y + 4$

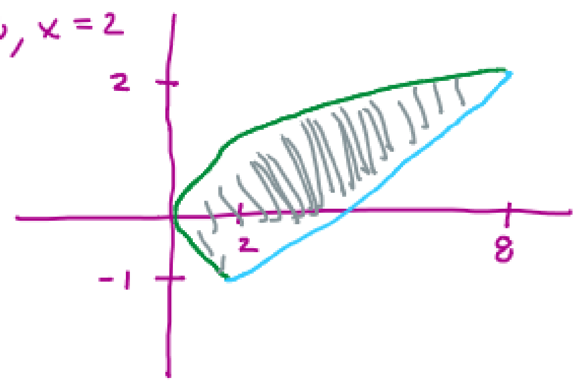
INTERSECTION

$2y^2 - 4 = 2y \Rightarrow 2y^2 - 2y - 4 = 0 \Rightarrow 2(y^2 - y - 2) = 0$

$2(y-2)(y+1) = 0$

$y = +2,\ y = -1$

$x = 8,\ x = 2$



$\int_{-1}^{2} 2y + 4 - 2y^2\,dy$

$= y^2 + 4y - \frac{2}{3}y^3 \Big|_{-1}^{2}$

$= \left(4 + 8 - \frac{16}{3}\right) - \left(1 - 4 + \frac{2}{3}\right)$

$= 12 - \frac{16}{3} - \left(-3 + \frac{2}{3}\right)$

$= 15 - \frac{18}{3} = 15 - 6 = \boxed{9}$

OR

$\int_0^2 \sqrt{\frac{x}{2}} - -\sqrt{\frac{x}{2}}\,dx$

$+ \int_2^8 \sqrt{\frac{x}{2}} - \left(\frac{x}{2} - 2\right)dx$

Area = $\boxed{9}$

4. (12 pts) The two parts below are NOT related.

(a) A particle is moving along the x-axis. The acceleration of the particle at time t seconds is given by

$$a(t) = 6t + 6 \text{ m/sec}^2,$$

and its initial velocity is $v(0) = -9$ m/sec. What is the **total distance** that the particle travels from $t = 0$ to $t = 3$ seconds?

$v(t) = 3t^2 + 6t - 9$

$3(t^2 + 2t - 3) = 0$

$3(t-1)(t+3) = 0$

$t = 1, \; t = -3$

$\int_0^3 |3t^2 + 6t - 9| \, dt$

$\int_0^1 3t^2 + 6t - 9 \, dt = t^3 + 3t^2 - 9t \Big|_0^1 = (1 + 3 - 9) - 0 = -5$

$\int_1^3 3t^2 + 6t - 9 \, dt = t^3 + 3t^2 - 9t \Big|_1^3 = (27 + 27 - 27) - (1 + 3 - 9) = 27 - (-5) = 32$

Total Distance = 37 meters

(b) A tomato is *dropped* from the top of a building. It hits the ground (next to your math instructor) with a downward velocity of 112 feet/sec. Assume the tomato accelerates at a constant 32 feet/sec^2 downward. Find the function for the height, $h(t)$ of the tomato t seconds after being dropped and give the height of the building? (*Hint: The time the tomato hits the ground is an unknown, label and find it.*)

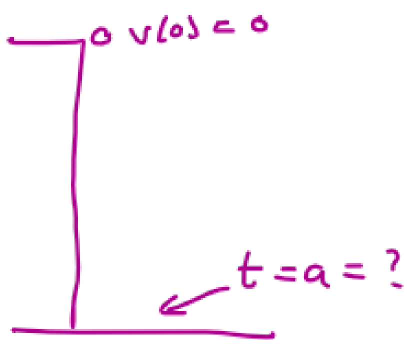
$a(t) = -32$

$v(t) = -32t + C \leftarrow C = 0$, $v(0) = 0$

$h(t) = -16t^2 + D$

GIVEN: $h(a) = 0 \rightarrow -16a^2 + D = 0$

$v(a) = -112 \rightarrow -32a = -112$



$\Rightarrow a = \frac{-112}{-32} = 3.5$

$\Rightarrow -16(3.5)^2 + D = 0$

$D = 16(3.5)^2 = 196$

$h(t) = -16t^2 + 196$

Building Height = 196 feet

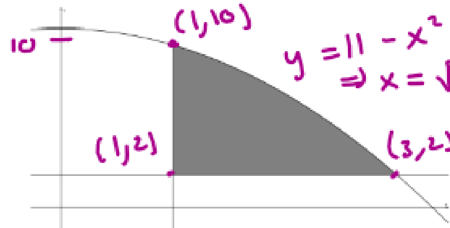
5. (12 pts) For all parts below, consider the region R that is bounded on the left by $x = 1$, bounded on the bottom by $y = 2$ and bounded on the top by $y = 11 - x^2$ (shown below).

(a) Set up BOTH the integrals (using dx and dy) that represent the area of this region. Include the correct bounds. DO NOT EVALUATE.

INTERSECTIONS:

$y = 11 - x^2$ & $x = 1 \Rightarrow y = 10$

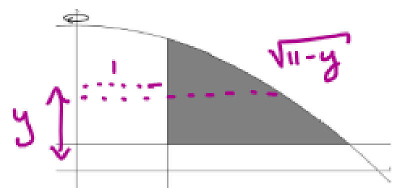
$y = 11 - x^2$ & $y = 2 \Rightarrow x = \pm 3$



Area set up (using dx) = $\int_1^3 11 - x^2 - 2\,dx = \int_1^3 9 - x^2\,dx$

Area set up (using dy) = $\int_2^{10} \sqrt{11 - y} - 1\,dy$

(b) Set up (but DO NOT EVALUATE) an integral for the VOLUME of the solid obtained by rotating R about the y-axis. Carefully include correct bounds and integrands (expect at least -2 per error, even small errors, so write your answers carefully!)



VOLUME = $\int_2^{10} \pi\left(\sqrt{11-y}\right)^2 - \pi(1)^2\,dy = \pi\int_2^{10} 11 - y - 1\,dy = \pi\int_2^{10} 10 - y\,dy$

(c) Set up (but DO NOT EVALUATE) an integral for the VOLUME of the solid obtained by rotating R about the horizontal line $y = 10$.

INNER = $10 - (11 - x^2) = x^2 - 1$

OUTER = $10 - 2 = 8$

VOLUME = $\int_1^3 \pi(8)^2 - \pi(x^2 - 1)^2\,dx$