

Finish 9.4 (Mixing, Air Resistance and Summary)

Mixing Problems:

“Rate of change equals rate IN minus rate OUT.”

Example 1:

A 12 Liter vat contains 7 kg of salt initially.

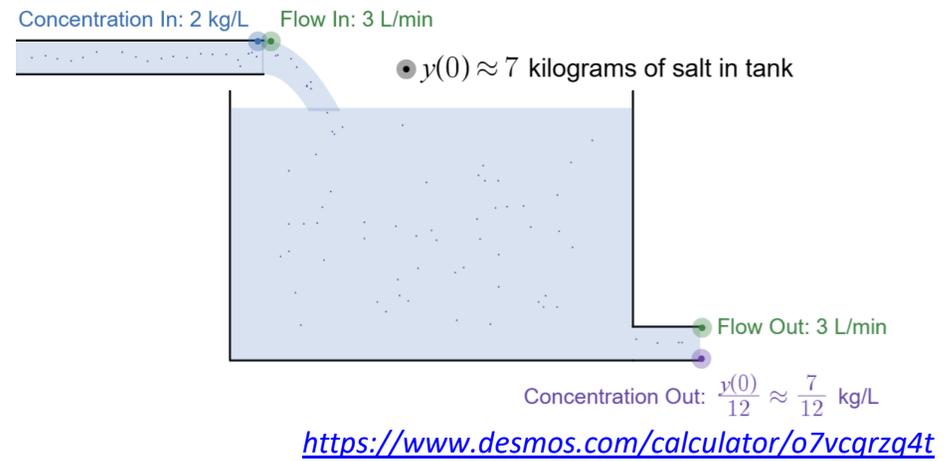
A pipe pumps in salt water (brine) at 3 L/min with a concentration of 2 kg/L of salt.

The mixture drains at 3 L/min.

The vat is well mixed.

Let $y(t)$ = “kg of salt in vat at time t ”.

- Find $y(t)$.
- Find the limit of $y(t)$ as $n \rightarrow \infty$.



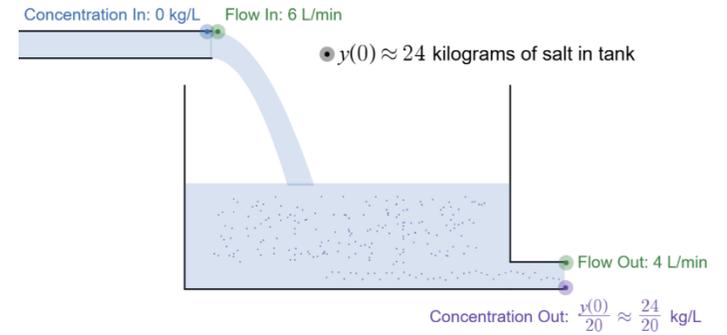
Mixing Problem Summary

$$\begin{aligned}\frac{dy}{dt} &= \text{Rate In} - \text{Rate out} \\ &= \left(? \frac{\text{kg}}{\text{L}} \right) \left(? \frac{\text{L}}{\text{min}} \right) - \left(\frac{y}{V} \frac{\text{kg}}{\text{L}} \right) \left(? \frac{\text{L}}{\text{min}} \right)\end{aligned}$$

$$y(0) = ? \text{ kg}$$

Example 2: How is this different?

A 50 Liter tank currently has 20 Liters of water with 24 kg of dissolved salt. A pipe pumps in *pure water* at 6 L/min. The mixture drains at 4 L/min. Let $y(t)$ = “kg of salt in vat at time t ”.



<https://www.desmos.com/calculator/v5itrp8xt1>

4. Air Resistance:

A skydiver steps out of a plane 4,000 meters high with an initial velocity of 0 m/s.

The skydiver has a mass of 60 kg.

(Treat downward as positive).

Newton's 2nd Law says:

$$m \frac{d^2 y}{dt^2} = \text{sum of forces on the object}$$

Forces

Gravity:

$$F_g = mg = 60 \cdot 9.8 = 588 \text{ N}$$

Air resistance: *(linear drag model)*

$$F_d = -k v \text{ N}$$

Assume $k = 12$ is the drag constant



Final Layout:

Pages 1, 2: Integrals
Page 8, 9: Diff. equations
Pages 3-7: Other concepts...

- Foundations
 - Riemann sums (idea/notation)
 - What an integral represents
 - FTOC (part 1)
- Acceleration/Velocity
 - “Tomato” problems
 - Net Change / Total Change
- Volumes of Revolution
- Simpson’s/Trapezoid Rules
- Improper Integrals
- Other Applications
 - Average Value
 - Arc Length
 - ~~Center of Mass~~
 - Work