9.3: Separable Differential Equations

Entry Task: *(Motivation)*
Implicitly differentiate $x^2 + y^3 = 8$
and solve for $\frac{dy}{dx}$.

Idea: separate and integrate both sides.
Entry Task continued:
Find the *explicit* solution for $\frac{dy}{dx} = \frac{-2x}{3y^2}$
with $y(0) = 2$.

9.3: Separable Differential Equations

A **separable** differential equation can be written as:

$$\frac{dy}{dx} = f(x)g(y).$$

(or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ or $\frac{dy}{dx} = \frac{g(y)}{f(x)}$.)

Example 1

Find the explicit solution to

$\frac{dy}{dx} = \frac{x}{y^4}$ with $y(0) = 1$.

Example 2

Find the explicit solution to

$$\frac{dy}{dx} = \frac{x\sin(2x)}{3y} \quad \text{with } y(0) = -1.$$

Example 3

Find the explicit solution to

$$(x+1)\frac{dy}{dx} = \frac{x^2}{e^y} \quad \text{with } y(0) = 0.$$

Case Study: Law of Natural Growth

P(t) = population at year *t*,

$\frac{dP}{dt} =$ rate of change of the population

The law of natural growth assumes

$$\frac{dP}{dt} = kP,$$

for some constant *k*

(we call *k* the *<u>relative</u> growth rate*).

Assumption:

"The rate of growth of a population is proportional to the size of the population."

Find the explicit solution to

$$\frac{dP}{dt} = kP \text{ with } P(0) = P_0$$

Example 1 Populations Growth

- 500 bacteria are in a dish at t=0 hr.
- 8000 bacteria are in the dish at t=3 hr.

Assume the population grows at a rate proportional to its size.

Find the function, B(t), for the bacteria population with respect to time.

Example 2 Radio-Active Decay

- The *half-life* of cesium-137 is 30 years.
- We start with a 100-mg sample.

Assume the mass decays at a rate proportional to its size.

Find the function, m(t), for the mass with respect to time.

Example 3 Bank Account

- You invest \$10,000 into a savings account.
- In 3 years, you notice your balance is \$10,400.

The balance grows at a rate proportional to its size (i.e. *interest* is a percentage of the balance at any time). You also never make additional deposits or withdrawals.

Find the function, A(t), for the amount of money in the account with respect to time.