

How would you **start** these?

1. $\int \frac{x^2 + 7}{x^2(3 - x)} dx$

2. $\int \sqrt{x} \ln(x) dx$

3. $\int \frac{1}{(x^2 + 6x + 13)^{3/2}} dx$

4. $\int \tan^{-1}(x) dx$

5. $\int \sin^2(x) \cos^3(x) dx$

6. $\int \frac{1}{x^2 \sqrt{25 - x^2}} dx$

7. $\int \frac{\sqrt{x}}{x - 9} dx$

8. $\int \tan^4(x) \sec^4(x) dx$

9. $\int x \sqrt{4 - x} dx$

7.7 Approximating Integrals

We have learned how to integral some important situations.

But many, many, many integrals CANNOT be done with any of our methods.

So, in a great many applications, we have to approximate!

To approximate $\int_a^b f(x)dx$

1. Compute $\Delta x = \frac{b-a}{n}$.

Label the tick marks: $x_i = a + i\Delta x$

2. Use an approximation method, see below...

$$L_n = \Delta x [f(x_0) + f(x_1) + \cdots + f(x_{n-1})] \quad (\text{Left endpoint})$$

$$R_n = \Delta x [f(x_1) + f(x_2) + \cdots + f(x_n)] \quad (\text{Right endpoint})$$

$$M_n = \Delta x [f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)] \quad (\text{Midpoint})$$

New - Trapezoid Rule:

(all the "middle terms" are multiplied by 2)

$$T_n = \frac{1}{2} \Delta x [f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

New - Simpson's Rule: n must be even!

(alternating multiplying middle terms by 4 and 2)

$$S_n = \frac{1}{3} \Delta x [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Example: Using $n = 4$ subdivisions, estimate

$$\int_0^4 \sqrt{100 - x^3} dx$$

Step 1: $\Delta x = \frac{4-0}{4} = 1.$ $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4$

Step 2: Here is each method:

$$(1) \left[\sqrt{100 - (0)^3} + \sqrt{100 - (1)^3} + \sqrt{100 - (2)^3} + \sqrt{100 - (3)^3} \right] \approx 38.0855 = L_4$$

$$(1) \left[\sqrt{100 - (1)^3} + \sqrt{100 - (2)^3} + \sqrt{100 - (3)^3} + \sqrt{100 - (4)^3} \right] \approx 34.0855 = R_4$$

$$(1) \left[\sqrt{100 - (0.5)^3} + \sqrt{100 - (1.5)^3} + \sqrt{100 - (2.5)^3} + \sqrt{100 - (3.5)^3} \right] \approx 36.5672 = M_4$$

NEW – Trapezoid rule

$$\frac{1}{2} (1) \left[\sqrt{100 - (0)^3} + 2\sqrt{100 - (1)^3} + 2\sqrt{100 - (2)^3} + 2\sqrt{100 - (3)^3} + \sqrt{100 - (4)^3} \right]$$
$$T_4 \approx 36.0855$$

NEW – Simpson's rule

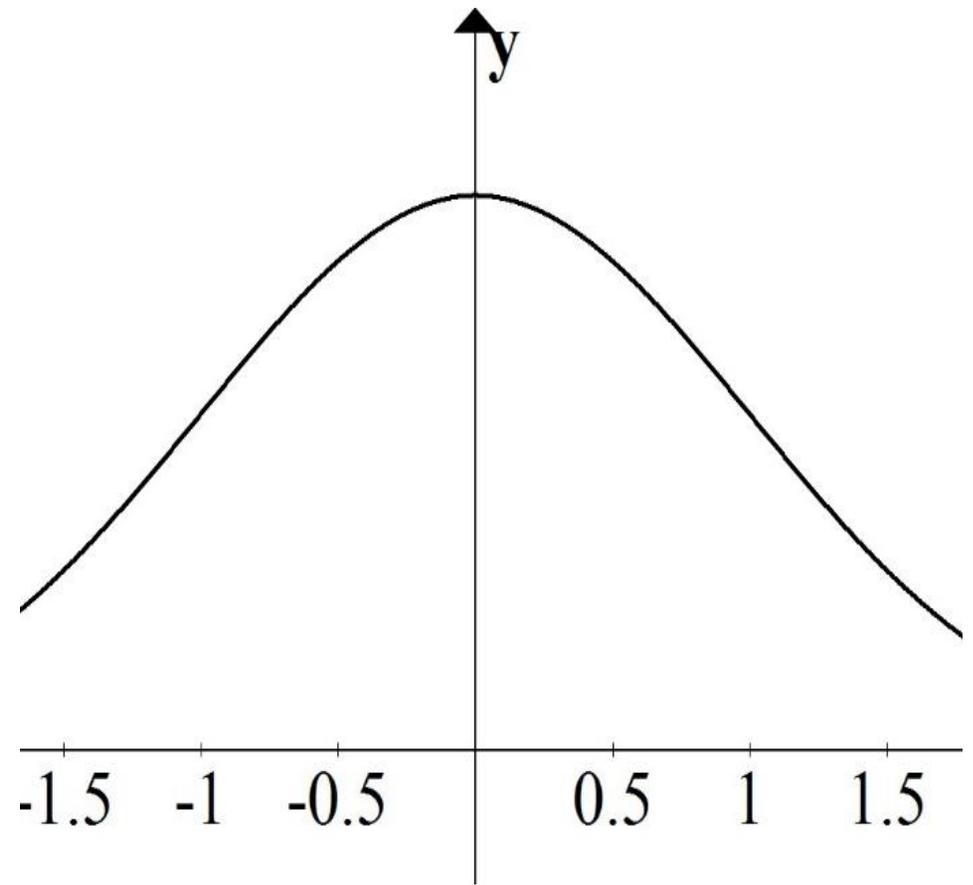
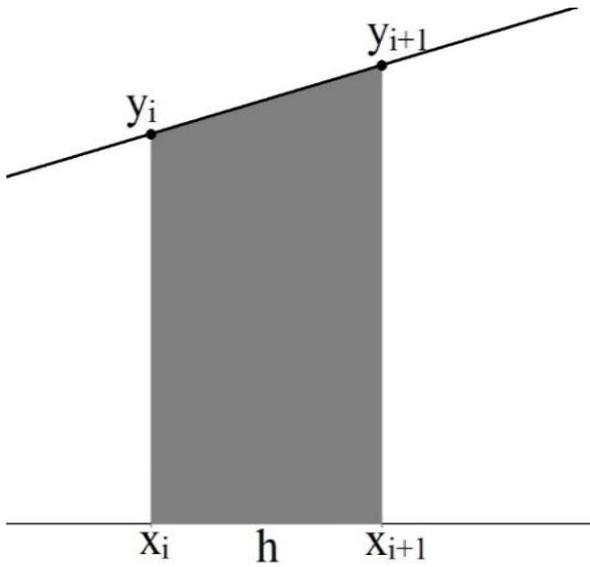
$$\frac{1}{3} \cdot (1) \left[\sqrt{100 - (0)^3} + 4\sqrt{100 - (1)^3} + 2\sqrt{100 - (2)^3} + 4\sqrt{100 - (3)^3} + \sqrt{100 - (4)^3} \right]$$
$$S_4 \approx 36.3863$$

“Actual” Value (to 8 places after the decimal) ≈ 36.40897795

7.7 Derivation Notes

Trapezoid Rule:

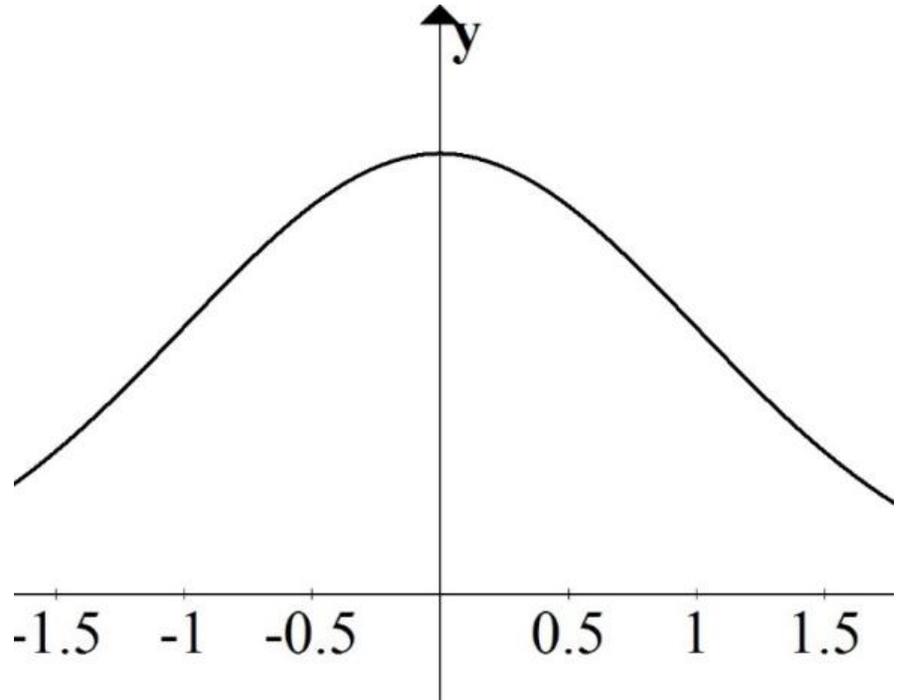
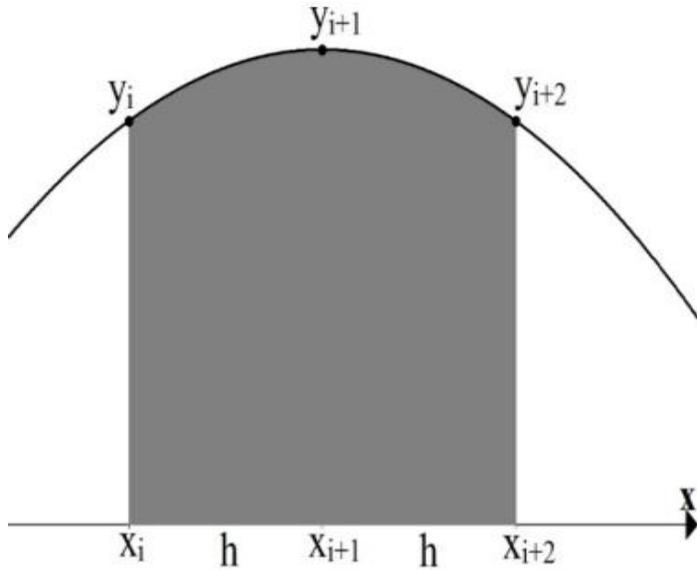
$$\text{Shaded Area} = \frac{h}{2}(y_i + y_{i+1})$$



Simpson's Rule:

The **parabola**, $y = ax^2 + bx + c$, thru the three indicated points, satisfies

$$\text{Shaded Area} = \frac{h}{3} (y_i + 4y_{i+1} + y_{i+2})$$



Example:

With $n = 4$, use both new methods to approximate
(just set up):

$$\frac{1}{\sqrt{2\pi}} \int_{-1}^1 e^{-\frac{1}{2}x^2} dx$$

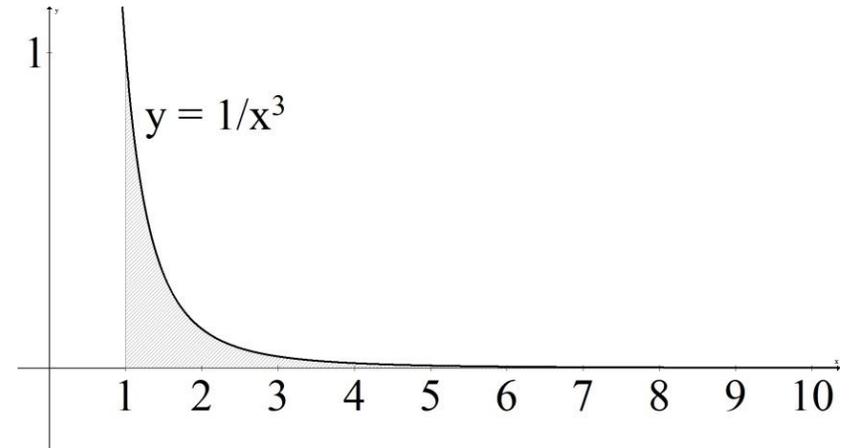
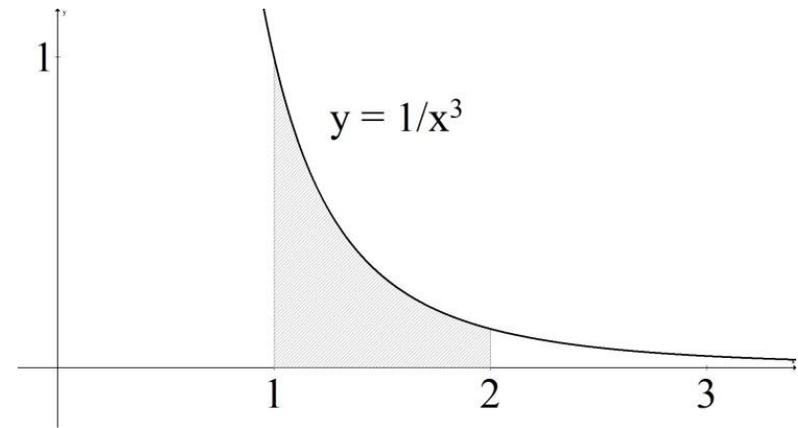
7.8 Improper Integrals (motivation and intro)

Motivation:

Consider the function $f(x) = \frac{1}{x^3}$.

Compute the area under the function...

1. ...from $x = 1$ to $x = t$
2. ...from $x = 1$ to $x = 10$
3. ...from $x = 1$ to $x = 100$



Def'n: Improper type 1

$$\int_a^{\infty} f(x)dx = \lim_{t \rightarrow \infty} \int_a^t f(x)dx$$

$$\int_{-\infty}^b f(x)dx = \lim_{t \rightarrow -\infty} \int_t^b f(x)dx$$

If the limit exists and is finite, then we say the integral *converges*. Otherwise, we say it *diverges*.

Example:

$$\int_1^{\infty} \frac{1}{x^3} dx =$$

Example:

$$\int_{-\infty}^2 e^{3x} dx =$$

Example:

$$\int_1^{\infty} \frac{1}{x} dx =$$

Example:

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$$

Def'n:

$$\int_{-\infty}^{\infty} f(x) dx = \lim_{r \rightarrow -\infty} \int_r^0 f(x) dx + \lim_{t \rightarrow \infty} \int_0^t f(x) dx$$

In this case, we say it *converges* only if both limits separately exist and are finite.