

Today: Finish Partial Fractions (7.4)
Summarize Integration (7.5)

Entry Task:

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx$$

7.4 Partial Fractions Method Summary

i) *Distinct Linear*: $\frac{x^2-3}{x(x-1)(x+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+4}$

ii) *Repeated Linear*: $\frac{5+2x}{(x+3)(x-2)^2} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

iii) *Irreducible Quad.*: $\frac{4x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$

Step 0: Reduce (if needed).

Step 1: Factor & write decomposition (see above)

Step 2: Solve for A, B, C...

Step 3: Integrate

Example:

$$\int \frac{x}{x^2 + 4x + 5} dx$$

7.5 Summary of how to integrate

A. Look for simplifications/substitutions

B. Products/Logs/Inverse Trig → BY PARTS

Sin/Cos/Tan/Sec combos → TRIG

Quadratic (under a radical) → TRIG SUB

Rational Function → PART. FRAC.

C. If nothing seems to work, substitution.

(u = inside, u = $\sqrt{\quad}$, u = trig, u = e^x)

Examples:

1. $\int e^{\sqrt{x}} dx$

$$2. \int \frac{\cos(x)}{4 - \sin^2(x)} dx$$

$$3. \int \frac{3}{x - 2\sqrt{x}} dx$$

$$4. \int e^x \cos(e^x) \sin^3(e^x) dx$$

How would you *start* these?

1. $\int \tan^3(x) \sec(x) dx$

2. $\int x^2 \ln(x) dx$

3. $\int x \sqrt{5 - x^2} dx$

4. $\int \frac{\sqrt{x^2 - 1}}{x^2} dx$

5. $\int \frac{x^2 + 1}{x^2 - 2x - 3} dx$

6. $\int x \tan^{-1}(x) dx$

7. $\int \frac{dx}{\sqrt{4x^2 + 8x - 12}} dx$